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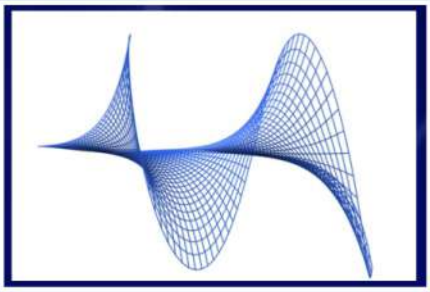

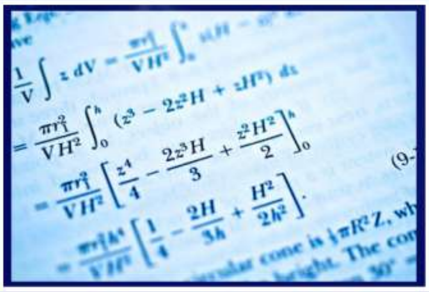

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Table of Contents

FUNDAMENTAL MATHEMATICS AND APPLICATIONS

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> Vol 6, No 1 (2023) (https://ejournal2.undip.ac.id/index.php/jfma
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CONSTRUCTION OF FUNDAMENTAL THEOREMS OF FRACTIONAL CALCULUS

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Abstract. *This paper discusses the theory of derivatives and integrals in the form of fractions with a particular order initiated by Lioville. Specifically, regarding the correlation between fractional derivatives and integrals, by examining definitions, determining the kernel function, and applying them to several examples, so a general formula will be obtained regarding the relationship between the two. This formula is the product of the fractional derivative of an order of a polynomial function of* m*-degree which is equal to the* $(n+1)^{th}$ *derivative of the related order fractional integral of a polynomial function of* m*-degree that the truth is proved by using Mathematical Induction.*
Keywords: *fractional derivative; fractional integral; Fundamental Theorem of Calculus.*

I. INTRODUCTION

One of the fundamental studies in mathematics, particularly in Calculus, is about derivatives and integrals. Lacroix developed a formula for the n^{th} derivatives of x^m in early 18th century [1], $\frac{d^n}{dx^n}x^m = \frac{\Gamma(m+1)}{\Gamma(m-n+1)}x^{m-n}$. Remember that $\Gamma(n) = \int_0^\infty x^{n-1}e^{-x}$, $n > 0$ [2] and $\Gamma(n+1) = n\Gamma(n)$ where $\Gamma(1) = 1$ [3]. Lacroix initiated replacing n with $\frac{1}{2}$ and $m = 1$ so it is obtained $\frac{d^{\frac{1}{2}}}{dx^{\frac{1}{2}}}x = \frac{2\sqrt{x}}{\sqrt{\pi}}$ and note that $\Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}$. Therefore, since then, studies related to fractional order derivatives have emerged. Several mathematicians who studied Fractional Calculus included Joseph Fourier, who initiated integral notation to denote derivatives with non-integer orders. Joseph Liouville and Bernhard Riemann established a general notation for fractional integrals. Mittag-Leffler, Grünwald, and Letnikov who greatly contributed to develop the Fractional Calculus [4]

Several researchers discussed and established the Fractional Calculus, H. Vit Danon was one of those who developed the theory, namely the Fundamental Theorem of Fractional Calculus[5], $D^{\frac{1}{2}}f(x) = DF^{-\frac{1}{2}}(x)$, Podlubny [6] discusses fractional partial differential functions, Hadamard [7] also states that derivative of non integer order of an analytic function can be found using the Taylor series. Gunawan et al [8]explain a method to find a continuous and well-defined function of two real variables on a surface that minimizes the fractional integral energy.

The focus of this research is to show the relationship between fractional derivatives and integrals, and construct the Fundamental Theorem of Fractional Calculus of a particular order of polynomial function of m-degree, and prove using Mathematical Induction. This article is expected to provide new knowledge for students and serve as a reference source for other researchers to apply and develop in related fields.

II. RIEMANN LIOUVILLE'S FRACTIONAL DERIVATIVES AND INTEGRALS

The Fundamental Theorem of Fractional Calculus is constructed using an analytical method. First, we will discuss the concept introduced by the French mathematician, Joseph Liouville, namely Liouville's Fractional Integral or Riemann-Liouville's Fractional Integral, which is a generalization of the Riemann-Stieltjes Integral for functions that have fractional derivatives [9]. Then, the corresponding kernel function is determined to obtain a generalized formula for fractional derivatives and integrals of order $n+\frac{k}{k+1}$and $n+\frac{k}{k+2}$ from polynomial functions of m-degree with some examples to be analyzed. However, in this article, we used the polynomial function[10] [11] $f(x)=x^m$. The final step is to create a general formula for the Fundamental Theorem of Fractional Calculus of order $n+\frac{k}{k+1}$ and $n+\frac{k}{k+2}$ that the truth is proved by using Mathematical Induction.

The fractional derivatives and integrals are obtained by extending the derivatives and integrals of integer order to the $q\in\mathbb{Q}$. In this article, Riemann Liouville's fractional derivatives and integrals can be identified by looking at the sign of each order, either positive or negative. If the sign is positive then it means fractional derivatives, whereas if the sign is negative then it means fractional integrals. First, Definition 1 and 2 will be given.

Definition 1 [12]*Let f and g Lebesgue Integrable on the interval* $(-\infty,\infty)$. *The convolution of function f and g can be denoted as $h=f*g$, where g is the kernel of convolution, and defined as*

$$h(x)=\int_{-\infty}^{\infty} f(t)g(x-t)\,dt.$$

Definition 2 *Let $f: G\to G'$ is a group homomorphism. The kernel of f can be denoted as Ker(f) and defined as Ker(f) $=\{x\in G|\ f(x)=e'\}$* [13][14]

Liouville defines fractional derivatives of order $\frac{1}{2}$, $\frac{1}{3}$, $\frac{2}{3}$, and fractional integrals order $-\frac{1}{2}$, $-\frac{1}{3}$, and $-\frac{2}{3}$ of the function f in [5][15]. We can apply the definitions to calculate the fractional derivatives of order $\frac{1}{2}$, $\frac{2}{3}$ and the integrals order $-\frac{1}{2}$ of $f(x)=x$, where $x\in[0,2]$, as follows

$$D^{\frac{1}{2}}x=\frac{1}{\Gamma\left(-\frac{1}{2}\right)}\int_0^x (x-t)^{-\frac{3}{2}}t\,dt=\frac{-1}{2\sqrt{\pi}}\left[2t(x-t)^{-\frac{1}{2}}+4(x-t)^{\frac{1}{2}}\right]_0^x=\frac{2x^{\frac{1}{2}}}{\sqrt{\pi}} \tag{1}$$

$$D^{\frac{2}{3}}x=\frac{1}{\Gamma\left(\frac{-2}{3}\right)}\int_0^x (x-t)^{-\frac{5}{3}}t\,dt=\frac{-\Gamma\left(\frac{2}{3}\right)}{\pi\sqrt{3}}\left[\frac{3}{2}t(x-t)^{-\frac{2}{3}}+\frac{9}{2}(x-t)^{\frac{1}{3}}\right]_0^x=\frac{9\Gamma\left(\frac{2}{3}\right)x^{\frac{1}{3}}}{2\pi\sqrt{3}} \tag{2}$$

In other side, we obtain fractional integrals of order $-\frac{1}{2}$

$$F^{-\frac{1}{2}}(x) = \frac{1}{\Gamma\left(\frac{1}{2}\right)}\int_0^x (x-t)^{-\frac{1}{2}} t\, dt = \frac{1}{\sqrt{\pi}}\left[-2t(x-t)^{\frac{1}{2}} - \frac{4}{3}(x-t)^{\frac{3}{2}}\right]_0^x = \frac{4x^{\frac{3}{2}}}{3\sqrt{\pi}} \tag{3}$$

The existence of the definition of Riemann-Liouville's Fractional Derivative, as the convolution of f and kernel, inspired the creation of several kernels for fractional derivatives of a particular order, which can be seen in Table 1.

Table 1. Kernel of Fractional Derivatives of Particular Orders

Form	Order	Kernel	Order	Kernel	Order	Kernel
I	$\frac{3}{4}$	$\frac{x^{-\frac{7}{4}}}{\Gamma\left(-\frac{3}{4}\right)}$	$\frac{7}{4}$	$\frac{x^{-\frac{11}{4}}}{\Gamma\left(-\frac{7}{4}\right)}$	$\frac{11}{4}$	$\frac{x^{-\frac{15}{4}}}{\Gamma\left(-\frac{11}{4}\right)}$
	$\frac{4}{5}$	$\frac{x^{-\frac{9}{5}}}{\Gamma\left(-\frac{4}{5}\right)}$	$\frac{9}{5}$	$\frac{x^{-\frac{14}{5}}}{\Gamma\left(-\frac{9}{5}\right)}$	$\frac{14}{5}$	$\frac{x^{-\frac{19}{5}}}{\Gamma\left(-\frac{14}{5}\right)}$
	$\frac{5}{6}$	$\frac{x^{-\frac{11}{6}}}{\Gamma\left(-\frac{5}{6}\right)}$	$\frac{11}{6}$	$\frac{x^{-\frac{17}{6}}}{\Gamma\left(-\frac{11}{6}\right)}$	$\frac{17}{6}$	$\frac{x^{-\frac{23}{6}}}{\Gamma\left(-\frac{17}{6}\right)}$
II	$\frac{2}{4}$	$\frac{x^{-\frac{6}{4}}}{\Gamma\left(-\frac{2}{4}\right)}$	$\frac{6}{4}$	$\frac{x^{-\frac{10}{4}}}{\Gamma\left(-\frac{6}{4}\right)}$	$\frac{10}{4}$	$\frac{x^{-\frac{14}{4}}}{\Gamma\left(-\frac{10}{4}\right)}$
	$\frac{3}{5}$	$\frac{x^{-\frac{8}{5}}}{\Gamma\left(-\frac{3}{5}\right)}$	$\frac{8}{5}$	$\frac{x^{-\frac{13}{5}}}{\Gamma\left(-\frac{8}{5}\right)}$	$\frac{13}{5}$	$\frac{x^{-\frac{18}{5}}}{\Gamma\left(-\frac{13}{5}\right)}$
	$\frac{4}{6}$	$\frac{x^{-\frac{10}{6}}}{\Gamma\left(-\frac{4}{6}\right)}$	$\frac{10}{6}$	$\frac{x^{-\frac{16}{6}}}{\Gamma\left(-\frac{10}{6}\right)}$	$\frac{16}{6}$	$\frac{x^{-\frac{22}{6}}}{\Gamma\left(-\frac{16}{6}\right)}$

Based on Table 1, the formula can be formed for fractional derivatives of several orders, such as order $\frac{3}{4}$ and $\frac{4}{6}$ in Definitions 3 and 4.

Definition 3 *Let f integrable function over the interval $[a,b]$. There exists $\frac{x^{-\frac{7}{4}}}{\Gamma\left(-\frac{3}{4}\right)}$ the kernel of fractional derivatives of order $\frac{3}{4}$, based on Table 1 and Definition 1 then the fractional derivatives of order $\frac{3}{4}$ can be denoted as $D^{\frac{3}{4}}f(x)$, defined as the convolution of f and kernel $\frac{x^{-\frac{7}{4}}}{\Gamma\left(-\frac{3}{4}\right)}$ over the interval $[a,x]$, such as*

$$D^{\frac{3}{4}}f(x) = \frac{1}{\Gamma\left(-\frac{3}{4}\right)}\int_a^x (x-t)^{-\frac{7}{4}} f(t)dt \tag{4}$$

Definition 4 *Let f integrable function over the interval [a,b]. There exists* $\frac{x^{-\frac{10}{6}}}{\Gamma\left(-\frac{4}{6}\right)}$ *the kernel of fractional derivatives of order* $\frac{4}{6}$*, based on Table 1 and Definition 1 then the fractional derivatives of order* $\frac{4}{6}$ *can be denoted as* $D^{\frac{4}{6}}f(x)$*, defined as the convolution of f and kernel* $\frac{x^{-\frac{10}{6}}}{\Gamma\left(-\frac{4}{6}\right)}$ *over the interval* $[a,x]$*, such as*

$$D^{\frac{4}{6}}f(x)=\frac{1}{\Gamma\left(-\frac{4}{6}\right)}\int_a^x (x-t)^{-\frac{10}{6}}f(t)dt \tag{5}$$

We will use the definition to calculate fractional derivatives of $f(x)=x$.

Example 1 Let $f(x)=x$, where $x\in[0,2]$. We will calculate the fractional derivatives of order $\frac{4}{6}$ and $\frac{3}{4}$, so we obtain

$$D^{\frac{4}{6}}f(x)=\frac{1}{\Gamma\left(\frac{-4}{6}\right)}\int_0^x (x-t)^{-\frac{10}{6}}udu=-\frac{\Gamma\left(\frac{4}{6}\right)}{\pi\sqrt{3}}\left[\frac{6}{4}t(x-t)^{-\frac{4}{6}}+\frac{18}{4}(x-t)^{\frac{2}{6}}\right]_0^x=\frac{18\Gamma\left(\frac{2}{3}\right)x^{\frac{2}{6}}}{4\pi\sqrt{3}}$$

$$D^{\frac{3}{4}}f(x)=\frac{1}{\Gamma\left(-\frac{3}{4}\right)}\int_0^x (x-t)^{-\frac{7}{4}}t\,dt=\frac{-3\Gamma\left(\frac{3}{4}\right)}{4\pi\sqrt{2}}\left[\frac{4}{3}t(x-t)^{-\frac{3}{4}}+\frac{16}{3}(x-t)^{\frac{1}{4}}\right]_0^x=\frac{16\Gamma\left(\frac{3}{4}\right)x^{\frac{1}{4}}}{4\pi\sqrt{2}}$$

Furthermore, the result in Example 1 (specifically order $\frac{4}{6}$) will be compared with order $\frac{2}{3}$ of $f(x)=x$ as we obtained previously. By (2), we have $\frac{9\Gamma\left(\frac{2}{3}\right)x^{\frac{1}{3}}}{2\pi\sqrt{3}}$ fractional derivatives of order $\frac{2}{3}$, while $\frac{18\Gamma\left(\frac{2}{3}\right)x^{\frac{2}{6}}}{4\pi\sqrt{3}}$ as the fractional derivatives of order $\frac{4}{6}$ and equal to $\frac{9\Gamma\left(\frac{2}{3}\right)x^{\frac{1}{3}}}{2\pi\sqrt{3}}$. Hence, we can state that the fractional derivatives of order $\frac{2}{3}$ and $\frac{4}{6}$ of $f(x)=x$ have the same result. So, this can be convincing that there is no contradiction between the definition in Table 1 and Definition 4 that has been made. Next, several examples of fractional derivatives with a particular order will be given based on the kernel search formed in Table 1.

Example 2 Let $f(x)=x$, where $x\in[0,2]$. We will calculate the fractional derivatives of order $\frac{7}{4}$, we obtain

$$D^{\frac{7}{4}}f(x)=\frac{1}{\Gamma\left(\frac{-7}{4}\right)}\int_0^x (x-t)^{-\frac{11}{4}}t\,dt=\frac{1}{\Gamma\left(\frac{-7}{4}\right)}\left[\frac{4}{7}t(x-t)^{-\frac{7}{4}}+\frac{16}{21}(x-t)^{-\frac{3}{4}}\right]_0^x=\frac{\Gamma\left(\frac{3}{4}\right)\sqrt[4]{x^{-3}}}{\pi\sqrt{2}}.$$

In similar way, we can calculate other examples as in Table 2 and 3. This can be used to construct the Fundamental Theorem of Fractional Calculus of a particular order.

Table 2. Example of Fractional Derivatives of Order Form I of $f(x) = x$ and $f(x) = x^2$

	Fractional Derivatives of $f(x)$					
	Order	x	x^2	Order	x	x^2
1	$\frac{1}{2}$	$\frac{2\sqrt{x}}{\sqrt{\pi}}$	$\frac{8\sqrt{x^3}}{3\sqrt{\pi}}$	$\frac{3}{2}$	$\frac{\sqrt{x^{-1}}}{\sqrt{\pi}}$	$\frac{4\sqrt{x}}{\sqrt{\pi}}$
2	$\frac{2}{3}$	$\frac{9\Gamma\left(\frac{2}{3}\right)\sqrt[3]{x}}{2\pi\sqrt{3}}$	$\frac{27\Gamma\left(\frac{2}{3}\right)\sqrt[3]{x^4}}{4\pi\sqrt{3}}$	$\frac{5}{3}$	$\frac{3\Gamma\left(\frac{2}{3}\right)\sqrt[3]{x^{-2}}}{2\pi\sqrt{3}}$	$\frac{9\Gamma\left(\frac{2}{3}\right)\sqrt[3]{x}}{\pi\sqrt{3}}$
3	$\frac{3}{4}$	$\frac{4\Gamma\left(\frac{3}{4}\right)\sqrt[4]{x}}{\pi\sqrt{2}}$	$\frac{32\Gamma\left(\frac{3}{4}\right)\sqrt[4]{x^5}}{5\pi\sqrt{2}}$	$\frac{7}{4}$	$\frac{\Gamma\left(\frac{3}{4}\right)\sqrt[4]{x^{-3}}}{\pi\sqrt{2}}$	$\frac{8\Gamma\left(\frac{3}{4}\right)\sqrt[4]{x}}{\pi\sqrt{2}}$
4	$\frac{4}{5}$	$\frac{5\Gamma\left(\frac{4}{5}\right)\sqrt[5]{x}}{\pi\csc\left(\frac{1}{5}\pi\right)}$	$\frac{50\Gamma\left(\frac{4}{5}\right)\sqrt[5]{x^6}}{6\pi\csc\left(\frac{1}{5}\pi\right)}$	$\frac{9}{5}$	$\frac{\Gamma\left(\frac{4}{5}\right)\sqrt[5]{x^{-4}}}{\pi\csc\left(\frac{1}{5}\pi\right)}$	$\frac{10\Gamma\left(\frac{4}{5}\right)\sqrt[5]{x}}{\pi\csc\left(\frac{1}{5}\pi\right)}$
⋮	⋮	⋮	⋮	⋮	⋮	⋮
k	$0 + \frac{k}{k+1}$	$D^{\left(0+\frac{k}{k+1}\right)}x$	$D^{\left(0+\frac{k}{k+1}\right)}x^2$	$1 + \frac{k}{k+1}$	$D^{\left(1+\frac{k}{k+1}\right)}x$	$D^{\left(1+\frac{k}{k+1}\right)}x^2$

Table 3. Example of Fractional Derivatives of Order Form II of $f(x) = x$ and $f(x) = x^2$

	Fractional Derivatives of $f(x)$					
	Order	x	x^2	Orde	x	x^2
1	$\frac{1}{3}$	$\frac{3x^{\frac{2}{3}}}{2\Gamma\left(\frac{2}{3}\right)}$	$\frac{9x^{\frac{5}{3}}}{5\Gamma\left(\frac{2}{3}\right)}$	$\frac{4}{3}$	$\frac{x^{-\frac{1}{3}}}{\Gamma\left(\frac{2}{3}\right)}$	$\frac{3x^{\frac{2}{3}}}{\Gamma\left(\frac{2}{3}\right)}$
2	$\frac{2}{4}$	$\frac{8x^{\frac{2}{4}}}{4\sqrt{\pi}}$	$\frac{32x^{\frac{6}{4}}}{12\sqrt{\pi}}$	$\frac{6}{4}$	$\frac{x^{-\frac{1}{2}}}{\sqrt{\pi}}$	$\frac{4x^{\frac{1}{2}}}{\sqrt{\pi}}$
3	$\frac{3}{5}$	$\frac{5\Gamma\left(\frac{3}{5}\right)x^{\frac{2}{5}}}{2\pi\csc\left(\frac{2}{5}\pi\right)}$	$\frac{25\Gamma\left(\frac{3}{5}\right)x^{\frac{7}{5}}}{7\pi\csc\left(\frac{2}{5}\pi\right)}$	$\frac{8}{5}$	$\frac{\Gamma\left(\frac{3}{5}\right)x^{-\frac{3}{5}}}{\pi\csc\left(\frac{2}{5}\pi\right)}$	$\frac{5\Gamma\left(\frac{3}{5}\right)x^{\frac{2}{5}}}{\pi\csc\left(\frac{2}{5}\pi\right)}$
4	$\frac{4}{6}$	$\frac{9\Gamma\left(\frac{2}{3}\right)x^{\frac{1}{3}}}{2\pi\sqrt{3}}$	$\frac{27\Gamma\left(\frac{2}{3}\right)x^{\frac{4}{3}}}{4\pi\sqrt{3}}$	$\frac{10}{6}$	$\frac{3\Gamma\left(\frac{2}{3}\right)x^{-\frac{2}{3}}}{2\pi\sqrt{3}}$	$\frac{9\Gamma\left(\frac{2}{3}\right)x^{\frac{1}{3}}}{\pi\sqrt{3}}$
⋮	⋮	⋮	⋮	⋮	⋮	⋮
k	$0 + \frac{k}{k+2}$	$D^{\left(0+\frac{k}{k+2}\right)}x$	$D^{\left(0+\frac{k}{k+2}\right)}x^2$	$1 + \frac{k}{k+2}$	$D^{\left(1+\frac{k}{k+2}\right)}x$	$D^{\left(1+\frac{k}{k+2}\right)}x^2$

III. CONSTRUCTION OF FUNDAMENTAL THEOREMS OF FRACTIONAL CALCULUS

The relationship between fractional derivatives and integrals is stated in the Fundamental Theorem of Fractional Calculus. The theorem states that the fractional derivative of order $\frac{1}{2}$ is equal to the derivative of fractional integrals of order $-\frac{1}{2}$. If Equations (3) and (1) are substituted into the Fundamental Theorem of Fractional Calculus of Order $\frac{1}{2}$, we will obtain

$$DF^{-\frac{1}{2}}(x) = \frac{d}{dx}\frac{4x^{\frac{3}{2}}}{3\sqrt{\pi}} = \frac{4\left(\frac{3}{2}\right)x^{\frac{3}{2}-1}}{3\sqrt{\pi}} = \frac{2x^{\frac{1}{2}}}{\sqrt{\pi}} = D^{\frac{1}{2}}x.$$

In particular, if the Fundamental Theorem of Fractional Calculus of Order $\frac{1}{2}$ is tried for higher order, we have

$$D^{\frac{3}{2}}f(x) = DD^{\frac{1}{2}}f(x) = D^2F^{-\frac{1}{2}}(x)$$
$$D^{\frac{5}{2}}f(x) = D^2D^{\frac{1}{2}}f(x) = D^3F^{-\frac{1}{2}}(x)$$
$$\vdots$$

More generally,

$$D^{n+\frac{1}{2}}f(x) = D^nD^{\frac{1}{2}}f(x) = D^{n+1}F^{-\frac{1}{2}}(x), n \geq 0.$$

In different order, we obtain

$$D^{n+\frac{1}{3}}f(x) = D^nD^{\frac{1}{3}}f(x) = D^{n+1}F^{-\frac{2}{3}}(x), n \geq 0$$
$$D^{n+\frac{2}{3}}f(x) = D^nD^{\frac{2}{3}}f(x) = D^{n+1}F^{-\frac{1}{3}}(x), n \geq 0.$$

If several examples of fractional derivatives in Table 2 and 3 are analyzed based on the Fundamental Theorem of Fractional Calculus of Order $\frac{1}{2}$, we have

$$D^{\frac{3}{4}}f(x) = D^{1-\frac{1}{4}}f(x) = DD^{-\frac{1}{4}}f(x) = DF^{-\frac{1}{4}}(x)$$
$$D^{\frac{4}{5}}f(x) = D^{1-\frac{1}{5}}f(x) = DD^{-\frac{1}{5}}f(x) = DF^{-\frac{1}{5}}(x)$$
$$D^{\frac{5}{6}}f(x) = D^{1-\frac{1}{6}}f(x) = DD^{-\frac{1}{6}}f(x) = DF^{-\frac{1}{6}}(x) \quad (6)$$

and

$$D^{\frac{2}{4}}f(x) = D^{1-\frac{2}{4}}f(x) = DD^{-\frac{2}{4}}f(x) = DF^{-\frac{2}{4}}(x)$$
$$D^{\frac{3}{5}}f(x) = D^{1-\frac{2}{5}}f(x) = DD^{-\frac{2}{5}}f(x) = DF^{-\frac{2}{5}}(x)$$
$$D^{\frac{4}{6}}f(x) = D^{1-\frac{2}{6}}f(x) = DD^{-\frac{2}{6}}f(x) = DF^{-\frac{2}{6}}(x) \quad (7)$$

Equation (6) and (7) initiate to establish the kernel of fractional integrals of particular order in Table 4.

By Table 4, we can calculate some examples in similar way as Example 2, which presented in Table 5.

Tabel 4. The Kernel of Fractional Integrals of Particular Orders

Order	$-\frac{1}{4}$	$-\frac{1}{5}$	$-\frac{1}{6}$	$-\frac{2}{4}$	$-\frac{2}{5}$	$-\frac{2}{6}$
Kernel	$\frac{x^{-\frac{3}{4}}}{\Gamma\left(\frac{1}{4}\right)}$	$\frac{x^{-\frac{4}{5}}}{\Gamma\left(\frac{1}{5}\right)}$	$\frac{x^{-\frac{5}{6}}}{\Gamma\left(\frac{1}{6}\right)}$	$\frac{x^{-\frac{2}{4}}}{\Gamma\left(\frac{2}{4}\right)}$	$\frac{x^{-\frac{3}{5}}}{\Gamma\left(\frac{2}{5}\right)}$	$\frac{x^{-\frac{4}{6}}}{\Gamma\left(\frac{2}{6}\right)}$

Tabel 5. Example of Fractional Integrals of Order Form I and II of $f(x) = x$ and $f(x) = x^2$

No.	Fractional Integral $f(x)$					
	Orde	x	x^2	Orde	x	x^2
1	$-\frac{1}{2}$	$\frac{4x^{\frac{3}{2}}}{3\sqrt{\pi}}$	$\frac{16x^{\frac{5}{2}}}{15\sqrt{\pi}}$	$-\frac{2}{3}$	$\frac{9x^{\frac{5}{3}}}{10\Gamma\left(\frac{2}{3}\right)}$	$\frac{27x^{\frac{8}{3}}}{40\Gamma\left(\frac{2}{3}\right)}$
2	$-\frac{1}{3}$	$\frac{27\Gamma\left(\frac{2}{3}\right)}{8\pi\sqrt{3}}x^{\frac{4}{3}}$	$\frac{81\Gamma\left(\frac{2}{3}\right)x^{\frac{7}{3}}}{28\pi\sqrt{3}}$	$-\frac{2}{4}$	$\frac{4x^{\frac{3}{2}}}{3\sqrt{\pi}}$	$\frac{16x^{\frac{5}{2}}}{15\sqrt{\pi}}$
3	$-\frac{1}{4}$	$\frac{16\Gamma\left(\frac{3}{4}\right)x^{\frac{5}{4}}}{5\pi\sqrt{2}}$	$\frac{128\Gamma\left(\frac{3}{4}\right)x^{\frac{9}{4}}}{45\pi\sqrt{2}}$	$-\frac{2}{5}$	$\frac{25\Gamma\left(\frac{3}{5}\right)x^{\frac{7}{5}}}{14\pi\csc\left(\frac{2}{5}\pi\right)}$	$\frac{250\Gamma\left(\frac{3}{5}\right)x^{\frac{12}{5}}}{168\pi\csc\left(\frac{2}{5}\pi\right)}$
4	$-\frac{1}{5}$	$\frac{25\Gamma\left(\frac{4}{5}\right)x^{\frac{6}{5}}}{6\pi\csc\left(\frac{1}{5}\pi\right)}$	$\frac{250\Gamma\left(\frac{4}{5}\right)x^{\frac{11}{5}}}{66\pi\csc\left(\frac{1}{5}\pi\right)}$	$-\frac{2}{6}$	$\frac{27\Gamma\left(\frac{2}{3}\right)}{8\pi\sqrt{3}}x^{\frac{4}{3}}$	$\frac{81\Gamma\left(\frac{2}{3}\right)x^{\frac{7}{3}}}{28\pi\sqrt{3}}$
⋮	⋮	⋮	⋮	⋮	⋮	⋮
k	$-\frac{1}{k+1}$	$F^{\left(-\frac{1}{k+1}\right)}(x)$	$F^{\left(-\frac{1}{k+1}\right)}(x)$	$-\frac{2}{k+2}$	$F^{\left(-\frac{2}{k+2}\right)}(x)$	$F^{\left(-\frac{2}{k+2}\right)}(x)$

By Table 5, Equation (6), and (7), we can generalize the Fundamental Theorem of Fractional Calculus of Order

Form I :

$$D^{\frac{k}{k+1}}f(x) = D^{1-\frac{1}{k+1}}f(x) = DD^{-\frac{1}{k+1}}f(x) = DF^{-\frac{1}{k+1}}(x),\ k \in \mathbb{N} \tag{8}$$

Form II :

$$D^{\frac{k}{k+2}}f(x) = D^{1-\frac{2}{k+2}}f(x) = DD^{-\frac{2}{k+2}}f(x) = DF^{-\frac{2}{k+2}}(x),\ k \in \mathbb{N} \tag{9}$$

Furthermore, if the product of fractional integrals of order $-\frac{1}{4}$ and $-\frac{2}{3}$ of x is differentiated twice, then

$$DF^{-\frac{1}{4}}(x) = \frac{d}{dx}\frac{16\Gamma\left(\frac{3}{4}\right)x^{\frac{5}{4}}}{5\pi\sqrt{2}} = \frac{4\Gamma\left(\frac{3}{4}\right)x^{\frac{1}{4}}}{\pi\sqrt{2}}$$

$$D^2F^{-\frac{1}{4}}(x) = DDF^{-\frac{1}{4}}(x) = \frac{d}{dx}\frac{4\Gamma\left(\frac{3}{4}\right)x^{\frac{1}{4}}}{\pi\sqrt{2}} = \frac{\Gamma\left(\frac{3}{4}\right)x^{-\frac{3}{4}}}{\pi\sqrt{2}}.$$

The fact, $\frac{4\Gamma\left(\frac{3}{4}\right)x^{\frac{1}{4}}}{\pi\sqrt{2}}$ and $\frac{\Gamma\left(\frac{3}{4}\right)x^{-\frac{3}{4}}}{\pi\sqrt{2}}$ are the product of fractional derivatives of order $\frac{3}{4}$ and $\frac{7}{4}$ of x. In different order we get

$$DF^{-\frac{2}{3}}(x) = \frac{d}{dx}\frac{9x^{\frac{5}{3}}}{10\Gamma\left(\frac{2}{3}\right)} = \frac{3x^{\frac{2}{3}}}{2\Gamma\left(\frac{2}{3}\right)}$$

$$D^2F^{-\frac{2}{3}}(x) = DDF^{-\frac{2}{3}}(x) = \frac{d}{dx}\frac{3x^{\frac{2}{3}}}{2\Gamma\left(\frac{2}{3}\right)} = \frac{x^{-\frac{1}{3}}}{\Gamma\left(\frac{2}{3}\right)},$$

From this result were obtained $\frac{3x^{\frac{2}{3}}}{2\Gamma\left(\frac{2}{3}\right)}$ and $\frac{x^{-\frac{1}{3}}}{\Gamma\left(\frac{2}{3}\right)}$ as fractional derivatives of order $\frac{1}{3}$ and $\frac{4}{3}$ of $f(x) = x$. This needs to be done to examine that (8) and (9) have been formulated and constructed correctly. Accordingly, we can state that the product of the fractional derivative of order of $n + \frac{k}{k+1}$ of polynomial function of m-degree is equal to the $(n+1)^{th}$ derivative of fractional integral of $-\frac{1}{k+1}$ of polynomial function of m-degree. Likewise, the product of the fractional derivative of order of $n + \frac{k}{k+2}$ of polynomial function of m-degree is equal to the $(n+1)^{th}$ derivative of fractional integral of $-\frac{1}{k+2}$ of polynomial function of m-degree, for arbitrary $n \geq 0, k \in \mathbb{N}$. Thus, it can be formulated as Propositions 1 and 2. Moreover, we can prove that validity by using Mathematical Induction [16].

Propositions 1 *if* $f(x) = x^m$, *then* $D^{n+\frac{k}{k+1}}f(x) = D^{n+1}F^{\left(-\frac{1}{k+1}\right)}(x)\ \forall n \geq 0,\ k \in \mathbb{N}$.

Proof:

Let $P(k)$: $D^{n+\frac{k}{k+1}}f(x) = D^{n+1}F^{\left(-\frac{1}{k+1}\right)}(x)$ for arbitrary $k \in \mathbb{N}$ and fix positive integer n.

a. Initial Step.
   If $k = 1$, then
   $P(1)$: $D^{n+\frac{1}{1+1}}f(x) = D^{n+\frac{1}{2}}f(x) = D^nD^{\frac{1}{2}}f(x) = D^{n+1}D^{-\frac{1}{2}}f(x) = D^{n+1}F^{-\frac{1}{2}}(x)$
   $\therefore P(1)$ is true
b. Inductive Step.
   Assume that $P(i)$: $D^{n+\frac{i}{i+1}}f(x) = D^{n+1}F^{\left(-\frac{1}{i+1}\right)}(x)$ is true.
   We want to prove that $P(i+1)$ is true, such that

$$\begin{aligned} P(i+1): D^{n+\frac{i+1}{(i+1)+1}}f(x) &= D^{n+\frac{i+1}{i+2}}f(x) \\ &= D^nD^{\frac{i+1}{i+2}}f(x) \\ &= D^nD^{\frac{i+2}{i+2}-\frac{1}{i+2}}f(x) \\ &= D^nD^{1-\frac{1}{i+2}}f(x) \\ &= D^{n+1}D^{-\frac{1}{i+2}}f(x) \\ &= D^{n+1}F^{\left(-\frac{1}{i+2}\right)}(x) \end{aligned}$$

   $\therefore P(i+1)$ is true

Hence a and b, $P(k)$: $D^{n+\frac{k}{k+1}}f(x) = D^{n+1}F^{\left(-\frac{1}{k+1}\right)}(x)$ is true for all $k \in \mathbb{N}, n \geq 0$. ■

Proposition 2. *If* $f(x) = x^m$, *then* $D^{n+\frac{k}{k+2}}f(x) = D^{n+1}F^{\left(-\frac{2}{k+2}\right)}(x)\ \forall n \geq 0,\ k \in \mathbb{N}$.

Proof:

Suppose $P(k)$: $D^{n+\frac{k}{k+2}}f(x) = D^{n+1}F^{\left(-\frac{2}{k+2}\right)}(x)$ for arbitrary $k \in \mathbb{N}$ and fix positive integer n.

a. Initial Step.
If $k = 1$, then
$P(1): D^{n+\frac{1}{1+2}}f(x) = D^{n+\frac{1}{3}}f(x) = D^{n}D^{1-\frac{2}{3}}f(x) = D^{n+1}D^{-\frac{2}{3}}f(x) = D^{n+1}F^{-\frac{2}{3}}(x)$
$\therefore P(1)$ is true.

b. Inductive Step.
Assume that $P(i)$: $D^{n+\frac{i}{i+2}}f(x) = D^{n+1}F^{\left(-\frac{2}{i+2}\right)}(x)$ is true.
We want to prove that $P(i+1)$ is true, such that

$$\begin{aligned} P(i+1): D^{n+\frac{i+1}{(i+1)+2}}f(x) &= D^{n+\frac{i+1}{i+3}}f(x) \\ &= D^{n}D^{\frac{i+3}{i+3}-\frac{2}{i+3}}f(x) \\ &= D^{n}D^{1-\frac{2}{i+3}}f(x) \\ &= D^{n+1}D^{-\frac{2}{i+3}}f(x) \\ &= D^{n+1}F^{\left(-\frac{2}{i+3}\right)}(x) \end{aligned}$$

$\therefore P(i+1)$ is true

Hence a and b, $P(k)$: $D^{n+\frac{k}{k+2}}f(x) = D^{n+1}F^{\left(-\frac{2}{k+2}\right)}(x)$ is true for all $k \in \mathbb{N},\ n \geq 0$. ■

IV. CONCLUSIONS

Based on the constructed Propositions 1 and 2, it can be concluded that there is a relationship between fractional derivatives and integrals. In particular, the product of the fractional derivative of order of $n + \frac{k}{k+1}$ and $n + \frac{k}{k+2}$ of polynomial function of m-degree which is equal to the $(n+1)^{th}$ derivative of fractional integral of $-\frac{1}{k+1}$ and $-\frac{1}{k+2}$ of polynomial function of m-degree, namely the Fundamental Theorem of Fractional Calculus of Order $n + \frac{k}{k+1}$ and $n + \frac{k}{k+2}, \forall n \geq 0, k \in \mathbb{N}$.

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8. **Positive aspects of the study and the manuscript:**

   Teorema dasar kalkulus fraksional untuk beberapa orde atau bentuk dikaji.

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   Flow artikel relatif sulit diikuti. Beberapa bagian pendahuluan redundant dengan existing results.

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    - Literature dapat diperkaya (sitasi minimal 15 sumber, utamakan paper-paper jurnal berreputasi)
    - Teorema 1 sudah well known. Teorema tersebut dapat direphrase menjadi literature review, atau dimasukkan ke dalam “Preliminaries”
    - Di paragraph terakhir Pendahuluan, disebutkan bahwa “fokus … korelasi…”. Saya tidak setuju dengan term korelasi karena istilah tersebut sudah terlalu mengakar di statistik, dan dapat diganti dengan “hubungan” atau “relasi”.
    - Apakah hasil-hasil pada Tabel 1-5 sudah well established? Jika iya, sebutkan sumbernya, dana saya kira tidak perlu dicantumkan disini, cukup disitasi jika dibutuhkan. Jika hasil tersebut baru, dapat dijabarkan salah satu contohnya.

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- Rumus (7) sudah well established, dan dapat dipindahkan ke "Preliminaries"

- Proposisi 1 dan 2 sama?

- Kalimat pertama pada kesimpulan perlu justifikasi lebih detail. Sebutkan dari rumus mana dan mana yng menjustifikasi hasil terbut.

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5. **Contribution to Science** : Fair

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7. **Overall Evaluation** : Average

8. **Positive aspects of the study and the manuscript:**

   Beberapa teorema dasar kalkulus dihasilkan.

9. **Negative aspects of the study and the manuscript:**

   Penulisan kurang baik dan pelru major revisions untuk memperbaiki penulisan.

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KONSTRUKSI TEOREMA DASAR KALKULUS FRAKSIONAL

Khairunnisa Fadhilla Ramdhania[1*], Rafika Sari[2], Rakhmi Khalida[3], dst

Abstract. This paper discusses the theory of derivatives and integrals in the form of fractions with a certain order initiated by Lioville. Specifically regarding the correlation between derivatives and fractional integrals, namely by examining definitions, determining function kernels, and applying them to several examples, so that a general formula regarding the correlation of the two will be obtained. This formula is the product of the fractional derivative of an order of a polynomial function of m-degree which is the same as the $n+1$ derivative of the related order fractional integral of a polynomial function of m-degree that the truth is proved by using Mathematical Induction.
Keywords: fractional derivative; fractional integral; Fundamental Theorem of Calculus.

Abstrak. Dalam artikel ini dibahas mengenai teori turunan dan integral dengan bentuk pecahan dengan orde tertentu yang digagas oleh Lioville. Khususnya tentang korelasi antara turunan dan integral fraksional, yakni dengan mengkaji definisi, menentukan kernel fungsi, dan menerapkan pada beberapa contoh, sehingga akan diperoleh rumus umum terkait korelasi keduanya. Rumus tersebut merupakan hasil turunan fraksional suatu orde dari fungsi polinom berderajat m sama dengan hasil turunan ke-$n+1$ dari integral fraksional orde terkait dari fungsi polinom berderajat m yang kebenarannya dibuktikan menggunakan Induksi Matematika.
Kata Kunci: turunan fraksional; integral fraksional; Teorema Dasar Kalkulus.

I. PENDAHULUAN

Salah satu kajian dalam ilmu matematika yang fundamental, khususnya dalam Kalkulus, adalah tentang turunan dan integral. Konsep turunan dipikirkan pada saat yang bersamaan oleh Newton dan Leibniz tahun 1665-1675. Pada tahun 1695 dalam sebuah surat yang ditulis oleh Leibniz kepada L'Hopital, ditujukan sebuah pertanyaan tentang turunan dengan orde bukan bilangan bulat. Pertanyaan tersebut dijawab dengan sebuah pertanyaan kembali oleh L'Hopital dengan turunan berorde $\frac{1}{2}$, yaitu berbentuk pecahan (fraction). Lacroix mengembangkan suatu formula untuk turunan ke n dari x^m pada awal abad 18, yaitu [1]

Teorema 1 *Misalkan* $f(x) = x^n$

$$\frac{d^n}{dx^n}x^m = \frac{m!}{(m-n)!}x^{m-n} = \frac{\Gamma(m+1)}{\Gamma(m-n+1)}x^{m-n} \quad (1)$$

dengan $n = 1, 2, .., m$.

Ingat bahwa $\Gamma(n) = \int_0^\infty x^{n-1}e^{-x}$, $n > 0$ [2] dan $\Gamma(n+1) = n\Gamma(n)$ dengan $\Gamma(1) = 1$ [3]. Kemudian, Lacroix menginisiasikan untuk mengganti n dengan $\frac{1}{2}$ dan $m = 1$ sehingga diperoleh $\frac{d^{\frac{1}{2}}}{dx^{\frac{1}{2}}}x = \frac{2\sqrt{x}}{\sqrt{\pi}}$ dan catat bahwa $\Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}$. Kemudian sejak saat itu mulailah bermunculan kajian-kajian yang terkait dengan turunan berorde pecahan. Beberapa matematikawan yang mengkaji tentang Kalkulus Fraksional di antaranya Joseph Fourier (1768-1830) yang menginisiasi penggunaan notasi integral untuk menyatakan turunan dengan orde bukan bilangan bulat. Joseph Liouville dan Bernhard Riemann membuat notasi umum untuk integral fraksional, serta ada Mittag-Leffler, Grünwald dan Letnikov yang sangat berkontribusi dalam berkembangan Kalkulus Fraksional [4]

Kini, banyak peneliti yang membahas dan mengaplikasikan Kalkulus Fraksional. H. Vic Dannon (2009) mengembangkan teori yakni tentang teorema dasar kalkulus fraksional [5]. Pada tahun 2009 Podlubny juga membahas fungsi diferensial parsial fraksional [6]. Gunawan dkk [7]menjelaskan metode untuk menemukan suatu fungsi real 2 variabel yang kontinu dan terdefinisi pada sebuah permukaan yang meminimumkan energi integral fraksional.

Fokus penelitian ini adalah untuk menunjukkan adanya korelasi antara turunan dan integral fraksional dan mengonstruksi Teorema Dasar Kalkulus Fraksional orde $n+\frac{k}{k+1}$ dan $n+\frac{k}{k+2}$ dari bentuk fungsi polinom berderajat m yang diharapkan dapat menjadi pengetahuan baru bagi mahasiswa dan sumber referensi bagi para peneliti lain untuk dapat diaplikasikan, serta dikembangkan sesuai dengan bidang yang berkaitan.

II. TURUNAN DAN INTEGRAL FRAKSIONAL RIEMANN LIOVILLE

Pengonstruksian Teorema Dasar Kalkulus Fraksional dilakukan dengan cara analitik. Pertama, akan dibahas mengenai konsep yang diperkenalkan oleh oleh matematikawan Prancis Joseph Liouville, yakni Integral Fraksional Liouville atau Integral Fraksional Riemann-Liouville, yang merupakan generalisasi Integral Riemann-Stieltjes untuk fungsi yang memiliki turunan pecahan [8]. Lalu, ditentukan fungsi kernel yang bersesuaian agar didapat formula untuk turunan dan integral fraksional dengan orde $n+\frac{k}{k+1}$ dan $n+\frac{k}{k+2}$ dari fungsi polinom berderajat m beserta contoh yang akan dianalisis. Namun, dalam artikel ini fungsi polinom yang digunakan adalah fungsi yang berbentuk $f(x)=x^m$. Langkah terakhir adalah membuat formula umum untuk Teorema Dasar Kalkulus Fraksional orde $n+\frac{k}{k+1}$ dan $n+\frac{k}{k+2}$ yang kebenarannya dibuktikan dengan Induksi Matematika.

Turunan dan integral fraksional diperoleh dengan memperluas turunan dan integral orde bilangan bulat, yakni menjadi $q\in\mathbb{Q}$. Turunan dan Integral Fraksional Riemann-Liouville pada artikel ini dapat dikenali dengan melihat tanda dari tiap-tiap orde, yakni apabila bertanda positif, maka yang dimaksud adalah turunan fraksional, sedangkan apabila bertanda negatif, maka yang dimaksud adalah integral fraksional. Sebelumnya, akan diberikan definisi 2 dan 3 yakni tentang konvolusi fungsi dan kernel fungsi.

Definisi 2 *Misal f dan g yang Lebesgue Integrable pada $(-\infty,\infty)$. Konvolusi dari f dan g dinotasikan $f*g$, ditulis $h=f*g$, didefinisikan sebagai*

$$h(x)=\int_{-\infty}^{\infty} f(t)g(x-t)\,dt$$

dimana $g(x)$ kernel konvolusi [9]

Definisi 3 *Misal $f: G\to G'$ adalah homomorfisma grup. Kernel dari f dinotasikan oleh Ker(f) dan didefinisikan Ker(f) $=\{x\in G|\ f(x)=e'\}$* [10].

Liouville mendefinisikan turunan fraksional orde $\frac{1}{2}$, $\frac{1}{3}$, dan $\frac{2}{3}$, serta integral fraksional orde $-\frac{1}{2}$, $-\frac{1}{3}$, dan $-\frac{2}{3}$ dari fungsi f di dalam Tabel 1 dan Tabel 2. Misalkan f *integrable* pada $[a,b]$.

Turunan dan integral fraksional didefinisikan sebagai konvolusi dari f dan kernel pada interval $[a,x]$, sebagai berikut.

Tabel 1. Definisi Turunan Fraksional Riemann-Liouville

Orde	Notasi	Kernel	Konvolusi	Formula
$\frac{1}{2}$	$\frac{d^{\frac{1}{2}}f(x)}{d(x-a)^{\frac{1}{2}}} = D^{\frac{1}{2}}f(x)$	$\frac{x^{-\frac{3}{2}}}{\Gamma\left(-\frac{1}{2}\right)}$	$\frac{x^{-\frac{3}{2}}}{\Gamma\left(-\frac{1}{2}\right)} * f(x)$	$\frac{1}{\Gamma\left(-\frac{1}{2}\right)}\int_a^x (x-t)^{-\frac{3}{2}}f(t)dt$
$\frac{1}{3}$	$\frac{d^{\frac{1}{3}}f(x)}{d(x-a)^{\frac{1}{3}}} = D^{\frac{1}{3}}f(x)$	$\frac{x^{-\frac{4}{3}}}{\Gamma\left(-\frac{1}{3}\right)}$	$\frac{x^{-\frac{4}{3}}}{\Gamma\left(-\frac{1}{3}\right)} * f(x)$	$\frac{1}{\Gamma\left(-\frac{1}{3}\right)}\int_a^x (x-t)^{-\frac{4}{3}}f(t)dt$
$\frac{2}{3}$	$\frac{d^{\frac{2}{3}}f(x)}{d(x-a)^{\frac{2}{3}}} = D^{\frac{2}{3}}f(x)$	$\frac{x^{-\frac{5}{3}}}{\Gamma\left(-\frac{2}{3}\right)}$	$\frac{x^{-\frac{5}{3}}}{\Gamma\left(-\frac{2}{3}\right)} * f(x)$	$\frac{1}{\Gamma\left(-\frac{2}{3}\right)}\int_a^x (x-t)^{-\frac{5}{3}}f(t)dt$

Tabel 2. Definisi Integral Fraksional Riemann-Liouville

Orde	Notasi	Kernel	Konvolusi	Formula
$-\frac{1}{2}$	$\frac{d^{-\frac{1}{2}}f(x)}{d(x-a)^{-\frac{1}{2}}} = D^{-\frac{1}{2}}f(x) = F^{-\frac{1}{2}}f(x)$	$\frac{x^{-\frac{1}{2}}}{\Gamma\left(\frac{1}{2}\right)}$	$\frac{x^{-\frac{1}{2}}}{\Gamma\left(\frac{1}{2}\right)} * f(x)$	$\frac{1}{\Gamma\left(\frac{1}{2}\right)}\int_a^x (x-t)^{-\frac{1}{2}}f(t)dt$
$-\frac{1}{3}$	$\frac{d^{-\frac{1}{3}}f(x)}{d(x-a)^{-\frac{1}{3}}} = D^{-\frac{1}{3}}f(x) = F^{-\frac{1}{3}}f(x)$	$\frac{x^{-\frac{2}{3}}}{\Gamma\left(\frac{1}{3}\right)}$	$\frac{x^{-\frac{2}{3}}}{\Gamma\left(\frac{1}{3}\right)} * f(x)$	$\frac{1}{\Gamma\left(\frac{1}{3}\right)}\int_a^x (x-t)^{-\frac{2}{3}}f(t)dt$
$-\frac{2}{3}$	$\frac{d^{-\frac{2}{3}}f(x)}{d(x-a)^{-\frac{2}{3}}} = D^{-\frac{2}{3}}f(x) = F^{-\frac{2}{3}}f(x)$	$\frac{x^{-\frac{1}{3}}}{\Gamma\left(\frac{2}{3}\right)}$	$\frac{x^{-\frac{1}{3}}}{\Gamma\left(\frac{2}{3}\right)} * f(x)$	$\frac{1}{\Gamma\left(\frac{2}{3}\right)}\int_a^x (x-t)^{-\frac{1}{3}}f(t)dt$

Tabel 1 dan 2 dapat diaplikasikan untuk mencari turunan fraksional orde $\frac{1}{2},\frac{2}{3}$ dan integral orde $-\frac{1}{2}$ untuk $f(x)=x$ dengan $x\in[0,2]$, sebagai berikut

$$D^{\frac{1}{2}}x = \frac{1}{\Gamma\left(-\frac{1}{2}\right)}\int_0^x (x-t)^{-\frac{3}{2}}t\,dt = \frac{-1}{2\sqrt{\pi}}\left[2t(x-t)^{-\frac{1}{2}} + 4(x-t)^{\frac{1}{2}}\right]_0^x = \frac{2x^{\frac{1}{2}}}{\sqrt{\pi}} \quad (2)$$

$$D^{\frac{2}{3}}x = \frac{1}{\Gamma\left(\frac{-2}{3}\right)}\int_0^x (x-t)^{-\frac{5}{3}}t\,dt = \frac{-\Gamma\left(\frac{2}{3}\right)}{\pi\sqrt{3}}\left[\frac{3}{2}t(x-t)^{-\frac{2}{3}} + \frac{9}{2}(x-t)^{\frac{1}{3}}\right]_0^x = \frac{9\Gamma\left(\frac{2}{3}\right)x^{\frac{1}{3}}}{2\pi\sqrt{3}} \quad (3)$$

Jika turunan tersebut dicari menggunakan Persamaan (1), maka akan diperoleh hasil yang sama pula.

Kemudian, integral fraksional orde $-\frac{1}{2}$ dapat dicari sebagai berikut

$$F^{-\frac{1}{2}}(x) = \frac{1}{\Gamma\left(\frac{1}{2}\right)}\int_0^x (x-t)^{-\frac{1}{2}}t\,dt = \frac{1}{\sqrt{\pi}}\left[-2t(x-t)^{\frac{1}{2}} - \frac{4}{3}(x-t)^{\frac{3}{2}}\right]_0^x = \frac{4x^{\frac{3}{2}}}{3\sqrt{\pi}} \quad (4)$$

Selain itu, adanya definisi turunan fraksional Riemann-Liouville pada Tabel 1 yang merupakan konvolusi dari f dan suatu kernel. mengilhami untuk dibuatnya beberapa kernel untuk turunan fraksional orde tertentu, yakni pada Tabel 3 berikut.

Tabel 3. Kernel Turunan Fraksional Orde Tertentu

Bentuk	Orde	Kernel	Orde	Kernel	Orde	Kernel
I	$\frac{3}{4}$	$\frac{x^{-\frac{7}{4}}}{\Gamma\left(-\frac{3}{4}\right)}$	$\frac{7}{4}$	$\frac{x^{-\frac{11}{4}}}{\Gamma\left(-\frac{7}{4}\right)}$	$\frac{11}{4}$	$\frac{x^{-\frac{15}{4}}}{\Gamma\left(-\frac{11}{4}\right)}$
	$\frac{4}{5}$	$\frac{x^{-\frac{9}{5}}}{\Gamma\left(-\frac{4}{5}\right)}$	$\frac{9}{5}$	$\frac{x^{-\frac{14}{5}}}{\Gamma\left(-\frac{9}{5}\right)}$	$\frac{14}{5}$	$\frac{x^{-\frac{19}{5}}}{\Gamma\left(-\frac{14}{5}\right)}$
	$\frac{5}{6}$	$\frac{x^{-\frac{11}{6}}}{\Gamma\left(-\frac{5}{6}\right)}$	$\frac{11}{6}$	$\frac{x^{-\frac{17}{6}}}{\Gamma\left(-\frac{11}{6}\right)}$	$\frac{17}{6}$	$\frac{x^{-\frac{23}{6}}}{\Gamma\left(-\frac{17}{6}\right)}$
II	$\frac{2}{4}$	$\frac{x^{-\frac{6}{4}}}{\Gamma\left(-\frac{2}{4}\right)}$	$\frac{6}{4}$	$\frac{x^{-\frac{10}{4}}}{\Gamma\left(-\frac{6}{4}\right)}$	$\frac{10}{4}$	$\frac{x^{-\frac{14}{4}}}{\Gamma\left(-\frac{10}{4}\right)}$
	$\frac{3}{5}$	$\frac{x^{-\frac{8}{5}}}{\Gamma\left(-\frac{3}{5}\right)}$	$\frac{8}{5}$	$\frac{x^{-\frac{13}{5}}}{\Gamma\left(-\frac{8}{5}\right)}$	$\frac{13}{5}$	$\frac{x^{-\frac{18}{5}}}{\Gamma\left(-\frac{13}{5}\right)}$
	$\frac{4}{6}$	$\frac{x^{-\frac{10}{6}}}{\Gamma\left(-\frac{4}{6}\right)}$	$\frac{10}{6}$	$\frac{x^{-\frac{16}{6}}}{\Gamma\left(-\frac{10}{6}\right)}$	$\frac{16}{6}$	$\frac{x^{-\frac{22}{6}}}{\Gamma\left(-\frac{16}{6}\right)}$

Berdasarkan Tabel 3, dapat dibentuk rumus untuk turunan fraksional beberapa orde. Di antaranya turunan fraksional orde $\frac{3}{4}$ dan $\frac{4}{6}$ yaitu:

Definisi 4 *Misal f integrable pada $[a,b]$. Pilih kernel turunan fraksional orde $\frac{3}{4}$ yaitu $\frac{x^{-\frac{7}{4}}}{\Gamma\left(-\frac{3}{4}\right)}$, dengan mengacu pada Tabel 1 dan Definisi 2 maka turunan fraksional orde $\frac{3}{4}$ dinotasikan oleh $D^{\frac{3}{4}}f(x)$, didefinisikan sebagai konvolusi dari f dan $\frac{x^{-\frac{7}{4}}}{\Gamma\left(-\frac{3}{4}\right)}$ pada interval $[a,x]$, yaitu*

$$D^{\frac{3}{4}}f(x) = \frac{1}{\Gamma\left(-\frac{3}{4}\right)}\int_a^x (x-t)^{-\frac{7}{4}}f(t)dt \tag{5}$$

Definisi 5 *Misal f integrable pada $[a,b]$. Pilih kernel turunan fraksional orde $\frac{4}{6}$ yaitu $\frac{x^{-\frac{10}{6}}}{\Gamma\left(-\frac{4}{6}\right)}$ dengan mengacu pada Tabel 1 dan Definisi 2 maka turunan fraksional orde $\frac{4}{6}$ dinotasikan oleh $D^{\frac{4}{6}}f(x)$, didefinisikan sebagai konvolusi dari f dan $\frac{x^{-\frac{10}{6}}}{\Gamma\left(-\frac{4}{6}\right)}$ pada interval $[a,x]$, yaitu*

$$D^{\frac{4}{6}}f(x) = \frac{1}{\Gamma\left(-\frac{4}{6}\right)}\int_a^x (x-t)^{-\frac{10}{6}}f(t)dt \tag{6}$$

Definisi yang telah dibuat akan digunakan untuk mencari contoh turunan fraksional sebagai berikut.

Contoh 1 Misalkan $f(x) = x$, dengan $x \in [0,2]$. Akan dicari turunan fraksional orde $\frac{4}{6}$ dan $\frac{3}{4}$, yaitu

$$D^{\frac{4}{6}} f(x) = \frac{1}{\Gamma\left(\frac{-4}{6}\right)} \int_0^x (x-t)^{-\frac{10}{6}} u du = -\frac{\Gamma\left(\frac{4}{6}\right)}{\pi\sqrt{3}} \left[\frac{6}{4} t (x-t)^{-\frac{4}{6}} + \frac{18}{4} (x-t)^{\frac{2}{6}}\right]_0^x = \frac{18\Gamma\left(\frac{2}{3}\right) x^{\frac{2}{6}}}{4\pi\sqrt{3}}$$

dan

$$D^{\frac{3}{4}} f(x) = \frac{1}{\Gamma\left(-\frac{3}{4}\right)} \int_0^x (x-t)^{-\frac{7}{4}} t \, dt = \frac{-3\Gamma\left(\frac{3}{4}\right)}{4\pi\sqrt{2}} \left[\frac{4}{3} t (x-t)^{-\frac{3}{4}} + \frac{16}{3} (x-t)^{\frac{1}{4}}\right]_0^x = \frac{16\Gamma\left(\frac{3}{4}\right) x^{\frac{1}{4}}}{4\pi\sqrt{2}}$$

Selanjutnya, hasil pencarian turunan fraksional di atas khususnya orde $\frac{4}{6}$ akan dibandingkan dengan orde $\frac{2}{3}$ dari $f(x) = x$ yang telah dicari sebelumnya. Berdasarkan Persamaan (2) didapat turunan fraksional orde $\frac{2}{3}$ dari x yaitu $\frac{9\Gamma\left(\frac{2}{3}\right) x^{\frac{1}{3}}}{2\pi\sqrt{3}}$, sedangkan turunan fraksional orde $\frac{4}{6}$ adalah $\frac{18\Gamma\left(\frac{2}{3}\right) x^{\frac{2}{6}}}{4\pi\sqrt{3}}$ yang sama dengan $\frac{9\Gamma\left(\frac{2}{3}\right) x^{\frac{1}{3}}}{2\pi\sqrt{3}}$. Dengan demikian, dapat dikatakan bahwa antara turunan fraksional orde $\frac{2}{3}$ dan orde $\frac{4}{6}$ dari $f(x) = x$ memiliki hasil yang sama. Hal tersebut mampu meyakinkan bahwa tidak ada kontradiksi antara definisi pada Tabel 1 dengan Definisi 5 yang telah dibuat. Berdasarkan hasil tersebut, akan diberikan beberapa contoh turunan fraksional dengan orde tertentu berdasarkan pencarian kernel yang terbentuk pada Tabel 3.

Tabel 4. Turunan Fraksional Orde Bentuk I

No.	$f(x)$					
	Orde	x	x^2	Orde	x	x^2
1	$\frac{1}{2}$	$\frac{2\sqrt{x}}{\sqrt{\pi}}$	$\frac{8\sqrt{x^3}}{3\sqrt{\pi}}$	$\frac{3}{2}$	$\frac{\sqrt{x^{-1}}}{\sqrt{\pi}}$	$\frac{4\sqrt{x}}{\sqrt{\pi}}$
2	$\frac{2}{3}$	$\frac{9\Gamma\left(\frac{2}{3}\right)\sqrt[3]{x}}{2\pi\sqrt{3}}$	$\frac{27\Gamma\left(\frac{2}{3}\right)\sqrt[3]{x^4}}{4\pi\sqrt{3}}$	$\frac{5}{3}$	$\frac{3\Gamma\left(\frac{2}{3}\right)\sqrt[3]{x^{-2}}}{2\pi\sqrt{3}}$	$\frac{9\Gamma\left(\frac{2}{3}\right)\sqrt[3]{x}}{\pi\sqrt{3}}$
3	$\frac{3}{4}$	$\frac{4\Gamma\left(\frac{3}{4}\right)\sqrt[4]{x}}{\pi\sqrt{2}}$	$\frac{32\Gamma\left(\frac{3}{4}\right)\sqrt[4]{x^5}}{5\pi\sqrt{2}}$	$\frac{7}{4}$	$\frac{\Gamma\left(\frac{3}{4}\right)\sqrt[4]{x^{-3}}}{\pi\sqrt{2}}$	$\frac{8\Gamma\left(\frac{3}{4}\right)\sqrt[4]{x}}{\pi\sqrt{2}}$
4	$\frac{4}{5}$	$\frac{5\Gamma\left(\frac{4}{5}\right)\sqrt[5]{x}}{\pi\csc\left(\frac{1}{5}\pi\right)}$	$\frac{50\Gamma\left(\frac{4}{5}\right)\sqrt[5]{x^6}}{6\pi\csc\left(\frac{1}{5}\pi\right)}$	$\frac{9}{5}$	$\frac{\Gamma\left(\frac{4}{5}\right)\sqrt[5]{x^{-4}}}{\pi\csc\left(\frac{1}{5}\pi\right)}$	$\frac{10\Gamma\left(\frac{4}{5}\right)\sqrt[5]{x}}{\pi\csc\left(\frac{1}{5}\pi\right)}$
⋮	⋮	⋮	⋮	⋮	⋮	⋮
k	$0+\frac{k}{k+1}$	$D^{\left(0+\frac{k}{k+1}\right)} x$	$D^{\left(0+\frac{k}{k+1}\right)} x^2$	$1+\frac{k}{k+1}$	$D^{\left(1+\frac{k}{k+1}\right)} x$	$D^{\left(1+\frac{k}{k+1}\right)} x^2$

Tabel 5. Turunan Fraksional Orde Bentuk II

No.	Turunan Fraksional $f(x)$					
	Orde	x	x^2	Orde	x	x^2
1	$\frac{1}{3}$	$\frac{3x^{\frac{2}{3}}}{2\Gamma\left(\frac{2}{3}\right)}$	$\frac{9x^{\frac{5}{3}}}{5\Gamma\left(\frac{2}{3}\right)}$	$\frac{4}{3}$	$\frac{x^{-\frac{1}{3}}}{\Gamma\left(\frac{2}{3}\right)}$	$\frac{3x^{\frac{2}{3}}}{\Gamma\left(\frac{2}{3}\right)}$
2	$\frac{2}{4}$	$\frac{8x^{\frac{2}{4}}}{4\sqrt{\pi}}$	$\frac{32x^{\frac{6}{4}}}{12\sqrt{\pi}}$	$\frac{6}{4}$	$\frac{x^{-\frac{1}{2}}}{\sqrt{\pi}}$	$\frac{4x^{\frac{1}{2}}}{\sqrt{\pi}}$
3	$\frac{3}{5}$	$\frac{5\Gamma\left(\frac{3}{5}\right)x^{\frac{2}{5}}}{2\pi\csc\left(\frac{2}{5}\pi\right)}$	$\frac{25\Gamma\left(\frac{3}{5}\right)x^{\frac{7}{5}}}{7\pi\csc\left(\frac{2}{5}\pi\right)}$	$\frac{8}{5}$	$\frac{\Gamma\left(\frac{3}{5}\right)x^{-\frac{3}{5}}}{\pi\csc\left(\frac{2}{5}\pi\right)}$	$\frac{5\Gamma\left(\frac{3}{5}\right)x^{\frac{2}{5}}}{\pi\csc\left(\frac{2}{5}\pi\right)}$
4	$\frac{4}{6}$	$\frac{9\Gamma\left(\frac{2}{3}\right)x^{\frac{1}{3}}}{2\pi\sqrt{3}}$	$\frac{27\Gamma\left(\frac{2}{3}\right)x^{\frac{4}{3}}}{4\pi\sqrt{3}}$	$\frac{10}{6}$	$\frac{3\Gamma\left(\frac{2}{3}\right)x^{-\frac{2}{3}}}{2\pi\sqrt{3}}$	$\frac{9\Gamma\left(\frac{2}{3}\right)x^{\frac{1}{3}}}{\pi\sqrt{3}}$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
k	$0+\frac{k}{k+2}$	$D^{\left(0+\frac{k}{k+2}\right)}x$	$D^{\left(0+\frac{k}{k+2}\right)}x^2$	$1+\frac{k}{k+2}$	$D^{\left(1+\frac{k}{k+2}\right)}x$	$D^{\left(1+\frac{k}{k+2}\right)}x^2$

Contoh-contoh pada Tabel 4 dan 5 berguna dalam pengonstruksian Teorema Dasar Kalkulus Fraksional orde tertentu.

III. KONSTRUKSI TEOREMA DASAR KALKULUS FRAKSIONAL

Korelasi antara turunan dan integral fraksional dinyatakan dalam Teorema Dasar Kalkulus (TDK) Fraksional, berikut ini merupakan TDK Fraksional orde $\frac{1}{2}$ [5]

Teorema Dasar Kalkulus Fraksional orde $\frac{1}{2}$

$$D^{\frac{1}{2}}f(x) = DF^{-\frac{1}{2}}(x) \tag{7}$$

Teorema tersebut menyatakan bahwa turunan fraksional orde $\frac{1}{2}$ sama dengan turunan dari integral fraksional orde $-\frac{1}{2}$. Apabila Persamaan (4) dan (2) disubstitusikan ke dalam Persamaan (7), akan diperoleh

$$DF^{-\frac{1}{2}}(x) = \frac{d}{dx}\frac{4x^{\frac{3}{2}}}{3\sqrt{\pi}} = \frac{4\left(\frac{3}{2}\right)x^{\frac{3}{2}-1}}{3\sqrt{\pi}} = \frac{2x^{\frac{1}{2}}}{\sqrt{\pi}} = D^{\frac{1}{2}}x.$$

Selanjutnya, jika secara khusus Persamaan (7) dicoba untuk orde yang lebih tinggi, maka

$$D^{\frac{3}{2}}f(x) = DD^{\frac{1}{2}}f(x) = D^2F^{-\frac{1}{2}}(x)$$

$$D^{\frac{5}{2}}f(x) = D^2D^{\frac{1}{2}}f(x) = D^3F^{-\frac{1}{2}}(x)$$

$$\vdots$$

Akibatnya, diperoleh

$$D^{n+\frac{1}{2}}f(x) = D^n D^{\frac{1}{2}}f(x) = D^{n+1}F^{-\frac{1}{2}}(x), n \geq 0$$

Begitu juga untuk Teorema Dasar Kalkulus Fraksional orde $\frac{1}{3}$ maupun $\frac{2}{3}$, berlaku

$$D^{n+\frac{1}{3}}f(x) = D^n D^{\frac{1}{3}}f(x) = D^{n+1}F^{-\frac{2}{3}}(x), n \geq 0$$
$$D^{n+\frac{2}{3}}f(x) = D^n D^{\frac{2}{3}}f(x) = D^{n+1}F^{-\frac{1}{3}}(x), n \geq 0.$$

Jika beberapa contoh turunan fraksional pada Tabel 5 dan 6 dianalisis dengan berdasar pada Persamaan (7), maka diperoleh

$$D^{\frac{3}{4}}f(x) = D^{1-\frac{1}{4}}f(x) = DD^{-\frac{1}{4}}f(x) = DF^{-\frac{1}{4}}(x)$$
$$D^{\frac{4}{5}}f(x) = D^{1-\frac{1}{5}}f(x) = DD^{-\frac{1}{5}}f(x) = DF^{-\frac{1}{5}}(x)$$
$$D^{\frac{5}{6}}f(x) = D^{1-\frac{1}{6}}f(x) = DD^{-\frac{1}{6}}f(x) = DF^{-\frac{1}{6}}(x) \quad (8)$$

serta

$$D^{\frac{2}{4}}f(x) = D^{1-\frac{2}{4}}f(x) = DD^{-\frac{2}{4}}f(x) = DF^{-\frac{2}{4}}(x)$$
$$D^{\frac{3}{5}}f(x) = D^{1-\frac{2}{5}}f(x) = DD^{-\frac{2}{5}}f(x) = DF^{-\frac{2}{5}}(x)$$
$$D^{\frac{4}{6}}f(x) = D^{1-\frac{2}{6}}f(x) = DD^{-\frac{2}{6}}f(x) = DF^{-\frac{2}{6}}(x) \quad (9)$$

Adanya persamaan di atas, menggagas terbentuknya kernel untuk integral fraksional dengan orde terkait pada Tabel 6.

Tabel 6. Kernel Integral Fraksional Orde Terkait

Orde	$-\frac{1}{4}$	$-\frac{1}{5}$	$-\frac{1}{6}$	$-\frac{2}{4}$	$-\frac{2}{5}$	$-\frac{2}{6}$
Kernel	$\frac{x^{-\frac{3}{4}}}{\Gamma\left(\frac{1}{4}\right)}$	$\frac{x^{-\frac{4}{5}}}{\Gamma\left(\frac{1}{5}\right)}$	$\frac{x^{-\frac{5}{6}}}{\Gamma\left(\frac{1}{6}\right)}$	$\frac{x^{-\frac{2}{4}}}{\Gamma\left(\frac{2}{4}\right)}$	$\frac{x^{-\frac{3}{5}}}{\Gamma\left(\frac{2}{5}\right)}$	$\frac{x^{-\frac{4}{6}}}{\Gamma\left(\frac{2}{6}\right)}$

Selanjutnya, dicari beberapa contoh integral fraksional dengan orde yang tercantum pada Tabel 6 dan orde terkait, yang disajikan dalam Tabel 7.

Tabel 7. Contoh integral fraksional orde tertentu dari $f(x)$

No.	Integral Fraksional $f(x)$					
	Orde	x	x^2	Orde	x	x^2
1	$-\frac{1}{2}$	$\frac{4x^{\frac{3}{2}}}{3\sqrt{\pi}}$	$\frac{16x^{\frac{5}{2}}}{15\sqrt{\pi}}$	$-\frac{2}{3}$	$\frac{9x^{\frac{5}{3}}}{10\Gamma\left(\frac{2}{3}\right)}$	$\frac{27x^{\frac{8}{3}}}{40\Gamma\left(\frac{2}{3}\right)}$
2	$-\frac{1}{3}$	$\frac{27\Gamma\left(\frac{2}{3}\right)}{8\pi\sqrt{3}}x^{\frac{4}{3}}$	$\frac{81\Gamma\left(\frac{2}{3}\right)x^{\frac{7}{3}}}{28\pi\sqrt{3}}$	$-\frac{2}{4}$	$\frac{4x^{\frac{3}{2}}}{3\sqrt{\pi}}$	$\frac{16x^{\frac{5}{2}}}{15\sqrt{\pi}}$

3	$-\frac{1}{4}$	$\frac{16\Gamma\left(\frac{3}{4}\right)x^{\frac{5}{4}}}{5\pi\sqrt{2}}$	$\frac{128\Gamma\left(\frac{3}{4}\right)x^{\frac{9}{4}}}{45\pi\sqrt{2}}$	$-\frac{2}{5}$	$\frac{25\Gamma\left(\frac{3}{5}\right)x^{\frac{7}{5}}}{14\pi\csc\left(\frac{2}{5}\pi\right)}$	$\frac{250\Gamma\left(\frac{3}{5}\right)x^{\frac{12}{5}}}{168\pi\csc\left(\frac{2}{5}\pi\right)}$
4	$-\frac{1}{5}$	$\frac{25\Gamma\left(\frac{4}{5}\right)x^{\frac{6}{5}}}{6\pi\csc\left(\frac{1}{5}\pi\right)}$	$\frac{250\Gamma\left(\frac{4}{5}\right)x^{\frac{11}{5}}}{66\pi\csc\left(\frac{1}{5}\pi\right)}$	$-\frac{2}{6}$	$\frac{27\Gamma\left(\frac{2}{3}\right)}{8\pi\sqrt{3}}x^{\frac{4}{3}}$	$\frac{81\Gamma\left(\frac{2}{3}\right)x^{\frac{7}{3}}}{28\pi\sqrt{3}}$
⋮	⋮	⋮	⋮	⋮	⋮	⋮
k	$-\frac{1}{k+1}$	$F^{\left(-\frac{1}{k+1}\right)}(x)$	$F^{\left(-\frac{1}{k+1}\right)}(x)$	$-\frac{2}{k+2}$	$F^{\left(-\frac{2}{k+2}\right)}(x)$	$F^{\left(-\frac{2}{k+2}\right)}(x)$

Berdasarkan Tabel 7, dapat dibuat perumuman dari Persamaan (8) dan (9) sehingga didapat

Bentuk I :

$$D^{\frac{k}{k+1}}f(x) = D^{1-\frac{1}{k+1}}f(x) = DD^{-\frac{1}{k+1}}f(x) = DF^{-\frac{1}{k+1}}(x),\ k \in \mathbb{N} \tag{10}$$

Bentuk II :

$$D^{\frac{k}{k+2}}f(x) = D^{1-\frac{2}{k+2}}f(x) = DD^{-\frac{2}{k+2}}f(x) = DF^{-\frac{2}{k+2}}(x),\ k \in \mathbb{N} \tag{11}$$

Kemudian, berdasarkan Tabel 7 dan Bentuk I juga II, apabila hasil integral fraksional orde $-\frac{1}{4}$ dan $-\frac{2}{3}$ dari x diturunkan dua kali secara bertahap, maka:

$$DF^{-\frac{1}{4}}(x) = \frac{d}{dx}\frac{16\Gamma\left(\frac{3}{4}\right)x^{\frac{5}{4}}}{5\pi\sqrt{2}} = \frac{4\Gamma\left(\frac{3}{4}\right)x^{\frac{1}{4}}}{\pi\sqrt{2}}$$

$$D^2F^{-\frac{1}{4}}(x) = DDF^{-\frac{1}{4}}(x) = \frac{d}{dx}\frac{4\Gamma\left(\frac{3}{4}\right)x^{\frac{1}{4}}}{\pi\sqrt{2}} = \frac{\Gamma\left(\frac{3}{4}\right)x^{-\frac{3}{4}}}{\pi\sqrt{2}}$$

dan $\frac{4\Gamma\left(\frac{3}{4}\right)x^{\frac{1}{4}}}{\pi\sqrt{2}}$ adalah hasil turunan fraksional orde $\frac{3}{4}$ dari x, sedangkan $\frac{\Gamma\left(\frac{3}{4}\right)x^{-\frac{3}{4}}}{\pi\sqrt{2}}$ adalah hasil turunan fraksional orde $\frac{7}{4}$ dari x, serta

$$DF^{-\frac{2}{3}}(x) = \frac{d}{dx}\frac{9x^{\frac{5}{3}}}{10\Gamma\left(\frac{2}{3}\right)} = \frac{3x^{\frac{2}{3}}}{2\Gamma\left(\frac{2}{3}\right)}$$

$$D^2F^{-\frac{2}{3}}(x) = DDF^{-\frac{2}{3}}(x) = \frac{d}{dx}\frac{3x^{\frac{2}{3}}}{2\Gamma\left(\frac{2}{3}\right)} = \frac{x^{-\frac{1}{3}}}{\Gamma\left(\frac{2}{3}\right)}$$

Dari hasil tersebut diperoleh $\frac{3x^{\frac{2}{3}}}{2\Gamma\left(\frac{2}{3}\right)}$ merupakan turunan fraksional orde $\frac{1}{3}$ dari $f(x) = x$, sedangkan $\frac{x^{-\frac{1}{3}}}{\Gamma\left(\frac{2}{3}\right)}$ merupakan turunan fraksional orde $\frac{4}{3}$ dari $f(x) = x$. Hal ini dilakukan untuk menguji apakah Persamaan (10) dan (11) sudah dirumuskan dengan tepat.

Berdasarkan hasil tersebut dan melakukan perumuman dapat dikatakan bahwa hasil turunan fraksional orde $n + \frac{k}{k+1}$ dari fungsi polinom berderajat-m sama dengan turunan ke $n+1$ dari hasil integral fraksional orde $-\frac{1}{k+1}$ dari fungsi polinom berderajat-m. Begitu juga dengan hasil turunan fraksional orde $n+\frac{k}{k+2}$ dari fungsi polinom berderajat-m sama dengan turunan ke $n+1$ dari hasil integral fraksional orde $-\frac{2}{k+2}$ dari fungsi polinom berderajat-m. Dengan demikian, dapat diformulasikan sebagai Proposisi 1 dan 2. Selain itu, pembuktiannya dengan menggunakan Induksi Matematika, yaitu misalkan $P(n)$ adalah pernyataan perihal bilangan bulat positif [11]. Untuk membuktikan bahwa $P(n)$ benar untuk semua bilangan bulat positif n, maka perlu menunjukkan bahwa:

a. $P(1)$ benar;

b. Jika $P(n)$ benar, maka $P(n+1)$ juga benar, untuk setiap $n \geq 1$.

Proposisi 1 Jika $f(x)=x^m$, maka $D^{n+\frac{k}{k+1}}f(x)=D^{n+1}F^{\left(-\frac{1}{k+1}\right)}(x)\ \forall n \geq 0,\ k \in \mathbb{N}$.

Bukti:

Misalkan $P(k)$: $D^{n+\frac{k}{k+1}}f(x)=D^{n+1}F^{\left(-\frac{1}{k+1}\right)}(x)$ untuk setiap $k \in \mathbb{N}$ dan $n>0$ tetap.

a. Basis Induksi.

Akan dibuktikan $P(1)$ benar

$P(1)$: $D^{n+\frac{1}{1+1}}f(x)=D^{n+\frac{1}{2}}f(x)=D^nD^{\frac{1}{2}}f(x)=D^{n+1}D^{-\frac{1}{2}}f(x)=D^{n+1}F^{-\frac{1}{2}}(x)$

$\therefore P(1)$ benar

b. Langkah Induktif. Misalkan $P(i)$, yaitu $P(i)$: $D^{n+\frac{i}{i+1}}f(x)=D^{n+1}F^{\left(-\frac{1}{i+1}\right)}(x)$ benar.

Akan dibuktikan $P(i+1)$ benar

$$\begin{aligned}P(i+1): D^{n+\frac{i+1}{(i+1)+1}}f(x) &= D^{n+\frac{i+1}{i+2}}f(x)\\ &= D^nD^{\frac{i+1}{i+2}}f(x)\\ &= D^nD^{\frac{i+2}{i+2}-\frac{1}{i+2}}f(x)\\ &= D^nD^{1-\frac{1}{i+2}}f(x)\\ &= D^{n+1}D^{-\frac{1}{i+2}}f(x)\\ &= D^{n+1}F^{\left(-\frac{1}{i+2}\right)}(x)\end{aligned}$$

$\therefore P(i+1)$ benar

Dari a dan b, terbukti bahwa $P(k)$: $D^{n+\frac{k}{k+1}}f(x)=D^{n+1}F^{\left(-\frac{1}{k+1}\right)}(x)$ adalah benar untuk setiap $n \geq 0,\ k \in \mathbb{N}$.

Proposisi 2. Jika $f(x)=x^m$, maka $D^{n+\frac{k}{k+2}}f(x)=D^{n+1}F^{\left(-\frac{2}{k+2}\right)}(x)\ \forall n \geq 0,\ k \in \mathbb{N}$.

Bukti:

Misalkan $P(k)$: $D^{n+\frac{k}{k+2}}f(x)=D^{n+1}F^{\left(-\frac{2}{k+2}\right)}(x)$ untuk setiap $k \in \mathbb{N}$ dan $n>0$ tetap.

a. Basis Induksi

Akan dibuktikan $P(1)$ benar

$P(1)$: $D^{n+\frac{1}{1+2}}f(x)=D^{n+\frac{1}{3}}f(x)=D^nD^{1-\frac{2}{3}}f(x)=D^{n+1}D^{-\frac{2}{3}}f(x)=D^{n+1}F^{-\frac{2}{3}}(x)$

$\therefore P(1)$ benar

b. Langkah Induktif. Misalkan $P(i)$, yaitu $P(i)$: $D^{n+\frac{i}{i+2}}f(x)=D^{n+1}F^{\left(-\frac{2}{i+2}\right)}(x)$ benar.

Akan dibuktikan $P(i+1)$ benar

$$P(i+1): D^{n+\frac{i+1}{(i+1)+2}} f(x) = D^{n+\frac{i+1}{i+3}} f(x)$$
$$= D^n D^{\frac{i+3}{i+3} - \frac{2}{i+3}} f(x)$$
$$= D^n D^{1-\frac{2}{i+3}} f(x)$$
$$= D^{n+1} D^{-\frac{2}{i+3}} f(x)$$
$$= D^{n+1} F^{\left(-\frac{2}{i+3}\right)}(x)$$

$\therefore P(i+1)$ benar

Dari a dan b, terbukti bahwa $P(k): D^{n+\frac{k}{k+2}} f(x) = D^{n+1} F^{\left(-\frac{2}{k+2}\right)}(x)$ adalah benar untuk setiap $n \geq 0,\ k \in \mathbb{N}$.

IV. KESIMPULAN

Berdasarkan hasil yang telah diperoleh, dapat disimpulkan bahwa terdapat korelasi antara turunan dan integral fraksional yaitu hasil turunan orde $n + \frac{k}{k+1}$ dari fungsi polinom berderajat-m sama dengan hasil turunan ke $n+1$ dari hasil integral fraksional orde $-\frac{1}{k+1}$ dari fungsi polinom berderajat m, begitu juga dengan hasil turunan fraksional orde $n + \frac{k}{k+2}$ dari fungsi polinom berderajat m sama dengan turunan $n+1$ dari hasil integral fraksional orde $-\frac{1}{k+2}$ dari fungsi polinom berderajat m, yang disebut sebagai Teorema Dasar Kalkulus Fraksional orde $n + \frac{k}{k+1}$ dan $n + \frac{k}{k+2}$, $\forall n \geq 0, k \in \mathbb{N}$.

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