

# The Simulation of Heat Propagation Rate in Smart Roasting Process Using Finite Difference Method

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## Abstract

In the industrial 4.0 era, there have been many developments in daily equipment that have been transformed into more sophisticated tools both in terms of how they are used, the efficiency of their work and the quality of the results obtained due to the digitization process that is easily accessed by many people. Therefore, As the early step, the paper discusses the computational modeling in the automatic cooking methods, namely the smart roaster system. The cooking method in the form of granules by using a roasting process, which generally utilizes heated sand, is interesting to learn since there are two mechanisms of heat propagation from the pan to the food object, namely the mechanism of heat propagation by conduction and radiation, but in this study, the heat propagation due to temporary convection was ignored. The first mechanism is accommodated by Fourier's Law, while the second is by Stefan-Boltzmann's Law. For simplification, the system is discussed in one dimension with the same grain size, but the object can have a different size. The composition of the pan walls, grains of sand, empty space, and food objects can be randomized so that various configurations can be created. Discussed how the process of heat propagation can occur in the roasting process and also radiation to the surrounding space on the top of the sand surface. The numerical analysis method is used with the finite difference method and the Matlab programming language for simulation.

**Keywords:** granular materials, roasting, heat propagation, finite difference.

## 1. Introduction

In industrial 4.0 era, many daily equipments have been transformed into more sophisticated tools both in terms of how they are used and the efficiency of their work. In cooking equipments, many companies have produced cooking equipment switch from conventional equipment to digital tools. For example, digitization of cooking utensils made by Sharp Company with Healsio Automatic Cookware products. By just putting all raw materials and then set the time for cooking, consumers can prepare it in the morning and then eat it when they get home from work. All features are made as easy as possible so that all consumers can later become chefs in their own homes. In addition, this product is supported by several features, such as automating heat which is able to regulate the level of heat in cooking. Steam and temperature sensor technology that is able to keep dishes from burning and overflowing and protect food from bacteria and fungi. There is also an automatic stirring feature that stirs and makes dishes blend well. Then the pre-set timer cooking feature allows consumers to set their own presentation time in accordance with the desired time. Healsio Automatic Cookware is produced with stainless material for the inner container and aluminum for the outer side. This product consumes 800 watts of electricity when cooking and when it warms only takes 50 to 70 watts.

Sand is a granular material, which is a material composed of smaller particles (Kreith and M.S.Bohn 2001). The mechanical and thermal properties of the particles are determined by the

thermal properties and interactions between the particles. Sand becomes an additional medium which is suitable for the roasting process because it can distribute the temperature evenly so that the roasted object does not burn. The heat propagation studied is only conduction and radiation that occurs in sand and roasted objects.

In this research, we will discuss how heat propagation in the roasting process uses sand media, where sand is granular material, a material composed of smaller particles whose mechanical properties are determined by interactions between the particles (Andrade et al. 2011). Two laws of heat propagation are used in this study, namely the Fourier conduction law (Kreith and M.S.Bohn 2001) and Stefan-Boltzmann law (Johnson 2012). Fourier's law discusses heat propagation in solids by conduction and Stefan-Boltzmann's law discusses heat propagation in a vacuum by radiation. The presence of air and its flow is ignored in this discussion. Problem solving is done numerically by using the finite difference method (difference up to) (Maulidi 2005). Furthermore, the process of visualisation is carried out the process of the rate of heat propagation in the process of cooking the roast system as a preliminary study of modeling of making intelligent stoves.

## 2. Research Methods

### 2.1 Tools and Materials

In this study, the roasted material was sand and the roasted object is a peanut. Roasting was done in a pan made of stainless steel. The position of the sand was at the bottom of the pan, while the position of the peanut is on the sand and allows it to be covered with sand. There was an empty gap between the sand formed by stirring the ingredients/baking material. The heat source was produced from an electric stove under the pan. Figure 1 shows the composition of this roasted material (Lubis, Sari, and Viridi 2018).

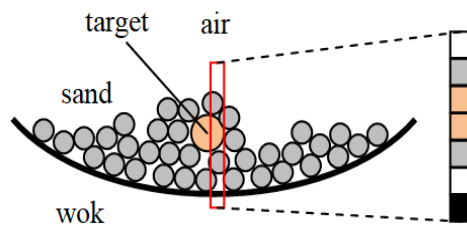


Figure 1. The composition of the materials in the roasting process.

Electric stove with resistance of heating coil  $R$  which is given voltage  $V$  will produce  $P_{coil}$  power formulated in equation (1).

$$P_{coil} = \frac{v_{in}^2}{R_{coil}} \quad (1)$$

The frying pan has  $m_{wok}$  mass in roasted process with  $c_{wok}$  type heat, initially has an initial  $T_{wok}(t)$  wok temperature. In time interval  $\Delta t$ , the temperature in the pan follows:

$$T_{wok}(t + \Delta t) = T_{wok}(t) + \frac{P_{coil} \Delta t}{m_{wok} c_{wok}} \quad (2)$$

### 2.2 One Dimension Heat Propagation

#### a. Fourier Law

Fourier's Law is a formulation to state the existence of conduction. Conduction is a heat dispersal event that only occurs in the particles of a material. In general, conduction occurs in solid media. Although the temperature gradient applies to solid media, heat will flow from the high temperature area to the low temperature area (Kreith and M.S.Bohn 2001). In the roasting process, heat moves from the pan to the sand, then the heat in the sand moves to the roasted object until the sand is on it.

The Fourier Law formulation in the roasting process begins with the statement of each symbol with the variables involved in the process. Solid materials, such as sand or cooking

objects (nuts), their area is expressed as  $A$ , length  $L$  and thermal conductivity  $\kappa$ . The heat delivered at each time unit  $\Delta t$  is formulated in equation (3).

$$Q = \Delta t A \kappa \left( \frac{T_H - T_L}{L} \right) \quad (3)$$

$T_H$  and  $T_L$  represent the temperature of each solid's end.  $T_H$  is the maximum temperature, whereas  $T_L$  is minimum temperature of each end.

#### b. Stefan-Boltzman Law

Stefan-Boltzmann's Law states the radiation process that occurs in the universe. Radiation is a heat propagation event without intermediates and takes place in a vacuum. The heat from the radiation field is emitted with electromagnetic waves (another alternative is photons) (Taylor, Fricke, and Becker 2011). Radiation occurs in the roasting process when heat travels between the cracks of sand grains and on the top sand surface. On the top surface of the sand, the radiated heat will go to the environment where the roasting process occurs. Stefan-Boltzmann's Law Formulation involves the variables involved in the roasting process to be symbolized. Assuming a particle with surface area  $A$  for each unit of time  $\Delta t$  will radiate the heat formulated in equation (4) below.

$$Q = A \Delta t \sigma \varepsilon (T^4 - T_{env}^4) \quad (4)$$

Suhu lingkungan dirumuskan dengan  $T_{env}$ , sedangkan  $\sigma$  dan  $\varepsilon$  masing-masing merupakan konstanta Stefan-Boltzmann dan emisivitas materi yang terkena radiasi. Konstanta Stefan-Boltzmann sebesar  $5,67 \times 10^{-8} \text{ W/m}^2\text{K}^4$ . (Sharma and Gujral 2011)

The ambient temperature is formulated with  $T_{env}$ , whereas  $\sigma$  and  $\varepsilon$  are Stefan-Boltzmann's constants and the emissivity of matter exposed to radiation, respectively. Stefan-Boltzmann constant is  $5.67 \times 10^{-8} \text{ W / m}^2\text{K}^4$  (Sharma and Gujral 2011).

#### c. The Law of Energy Conservation

Energy conservation in the roasting process occurs because of the heat propagation in each layer of the roast composition. The ambient temperature during the roasting process (or other events) is always constant. Therefore, a transient temperature  $T^\infty$  will be achieved. Thermal conduction is the direct microscopic exchange of the kinetic energy of particles through the boundary between two systems. When an object has a different temperature from the object or the surrounding environment, heat flows have the same temperature at a point of thermal equilibrium. Spontaneous heat transfer occurs from a high temperature to a low temperature, as explained by the second law of thermodynamics. Steady state conduction based on Fourier's law, is a form of conduction that occurs when the temperature difference in the conduction event takes place spontaneously. Then after equilibrium time, the spatial distribution of temperature in the conduction object does not change again. In steady state conduction, the amount of heat entering a part is equal to the amount of heat coming out. Transient conduction (at transient temperatures  $T^\infty$ ) arises when the temperature of an object changes as a function of time. Analysis on transient systems is more complex and is often used for applications of numerical analysis by computers. Thermal radiation occurs through a vacuum or transparent medium. Energy is transferred through photons in waves electromagnetic thermal radiation is the energy released by objects as electromagnetic waves, because there is a pile of thermal energy in all objects with temperatures above absolute zero, thermal radiation arises as a result of random displacement of atoms and molecules, because atoms and molecules are composed of particles charged (protons and electrons), their movements produce the release of electromagnetic radiation that carries energy.

### 2.3 The Numerical Finite Difference Method

The finite difference method is widely used to find the solutions of partial differential equations (PDP) problems. This is because the younger partial differential equation solutions are approached using the Taylor series. There are several different approaches to finite methods, i.e. forward, backward and central difference equations. In this study used advanced calculation. From the mathematical formulation of Fourier's law, the conduction rate of heat transfer in equation (3) is made into the 1-D partial differential equation form as follows.

$$Q = A \Delta t \sigma \varepsilon (T^4 - T_{env}^4) \tag{5}$$

where  $t > 0$  and  $0 < x < L$ ,

Equation (6) represents the one-dimensional conduction heat transfer equation since there is only one variable,  $x$ , with  $k$  positive constants. Variable  $t$  is the time variable, and  $U(x, t)$  is a function that depends on  $x$  and  $t$ . This equation, the Taylor  $U$  series ( $x, t + \Delta t$ ), is expressed at the  $t$  point (Tubert-Brohman et al. 2013).

$$U(x, t + \Delta t) = U(x, t) + U_t(x, t)\Delta t + \frac{1}{2}U_{tt}(x, t)\Delta t^2 + \frac{1}{6}U_{ttt}(x, t)\Delta t^3 + \dots \tag{6}$$

$$U(x, t + \Delta t) - U(x, t) = U_t(x, t)\Delta t + \frac{1}{2}U_{tt}(x, t)\Delta t^2 + \frac{1}{6}U_{ttt}(x, t)\Delta t^3 + \dots$$

$$\frac{U(x, t + \Delta t) - U(x, t)}{\Delta t} = U_t(x, t) + \frac{1}{2}U_{tt}(x, t)\Delta t + \frac{1}{6}U_{ttt}(x, t)\Delta t^2 + \dots$$

Because  $\frac{1}{2}U_{tt}(x, t)\Delta t + \frac{1}{6}U_{ttt}(x, t)\Delta t^2 + \dots$  is small number and can be ignored, equation (6) can be rewritten.

$$U_t(x, t) = \frac{U(x, t + \Delta t) - U(x, t)}{\Delta t} + O(\Delta t) \tag{6}$$

This means that forward advanced derivative formulations have a level one accuracy. With the same scheme, the radiation propagation rate will also be calculated.

#### 2.4. Simulation Modelling

This research used Matlab as programming language. In the beginning, a theoretical study was conducted to formulate mathematical equations that describe the dynamics of the grain material system in the one-dimensional heat propagation process of the roasted system. The mathematical formulation obtained is used to simulate the system using finite difference method.

As initial step, we divided layers of roasted composition based on conduction and radiation events that occur in each layer. The conduction layer used equation (3), while the radiation layer used equation (4). In equations (3) and (4), each variable represents the amount of heat energy generated from the conduction and radiation process. However, in the equation, the highest temperature and lowest temperature variables are known. In this study, we analyzed the temperature distribution at each layer of the roasted composition in order to get the expected variable from the equation becomes the lowest temperature (Manglik 2014). Figure 2 describes the description of the lowest temperature variable.

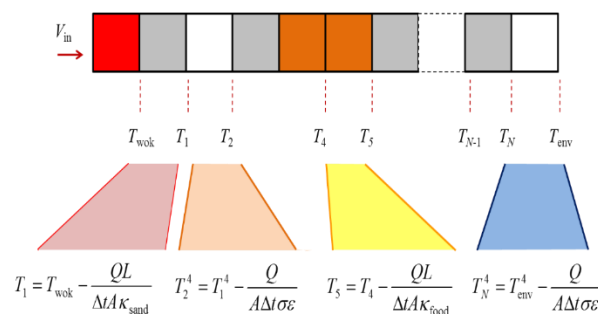


Figure 2. The composition of the formula for the conduction and radiation processes that occur at each layer of the roast process composition.

In the next step we determined the reduction of the intensity parameters in each layer of the roasted composition. The intensity parameter is useful for knowing how much heat energy is emitted per unit area and time on each layer surface. The formulation of the intensity parameter follows equation (7) below.

$$I = \frac{Q}{A \Delta t} \quad (7)$$

The intensity value is given the same at each layer. The limitation that needs to be considered is that the highest temperature difference with the lowest temperature must be greater than zero that is formulated with:  $T_H - T_L \gg 0$ . The intensity parameter value is  $I = 1$  J·s<sup>-1</sup>·m<sup>-2</sup>,  $\sigma = 5,670367 \times 10^{-8}$  W·m<sup>-2</sup>·K<sup>-4</sup>,  $\Delta t = 10^{-5}$  s,  $L = 1$  mm =  $10^{-3}$  m,  $V = 220$  V,  $R = 10$  Ω, dan  $T_{env} = 300$  K. (Sharma and Gujral 2011).

### 3. Results and Discussion

#### 3.1. Numerical Method Schema

Mathematical formulation based on the Fourier law equation of heat transfer rate by conduction 1-D is:  $Q = \Delta t A k \left( \frac{T_H - T_L}{L} \right)$  and the Stefan-Boltzmann 1-D radiation heat transfer rate equation is  $Q = A \Delta t \sigma \varepsilon (T^4 - T_{env}^4)$ ,

the two equations are converted into the form of 1-D partial differential equations such as equation (5). Equation (5) is called the one-dimensional conduction heat transfer equation because there is only one space  $x$  variable, with  $k$  positive constant,  $t$  is time, and  $U(x, t)$  is a function that depends on  $x$  and  $t$ . Then the Taylor Series was used for advanced difference derived formulations that have a value of error or accuracy  $O(\Delta t)$ , that is level one accuracy.

Next, we reduced the intensity parameters ( $I$ ),  $I = \frac{Q}{A \Delta t}$ . Where  $I$  is the amount of heat energy emitted by the broad union and time of each surface layer. The value of given  $I$  was similar for each layer. For row matrices in figure-2, the iteration process used the finite difference method in the heat propagation process of the roasted system as follows:

$$T_1 = T_{wok} - \frac{QL}{\Delta t A \kappa_{sand}} = T_{wok} - \frac{I}{\kappa_{sand}} \quad (8)$$

$$T_2^4 = T_1^4 - \frac{QL}{\Delta t A \sigma \varepsilon} = T_1^4 + \frac{I}{\sigma \varepsilon} \quad (9)$$

⋮

$$T_5 = T_4 - \frac{QL}{\Delta t A \kappa_{food}} = T_4 - \frac{I}{\kappa_{food}} \quad (10)$$

⋮

$$T_N^4 = T_{env}^4 - \frac{QL}{\Delta t A \sigma \varepsilon} = T_{env}^4 + \frac{I}{\sigma \varepsilon} \quad (11)$$

### 3.2 Simulation Flowchart

To facilitate the calculation of transfer or heat transfer rate in accordance with the above numerical scheme, Figure 3 shows a general flowchart.

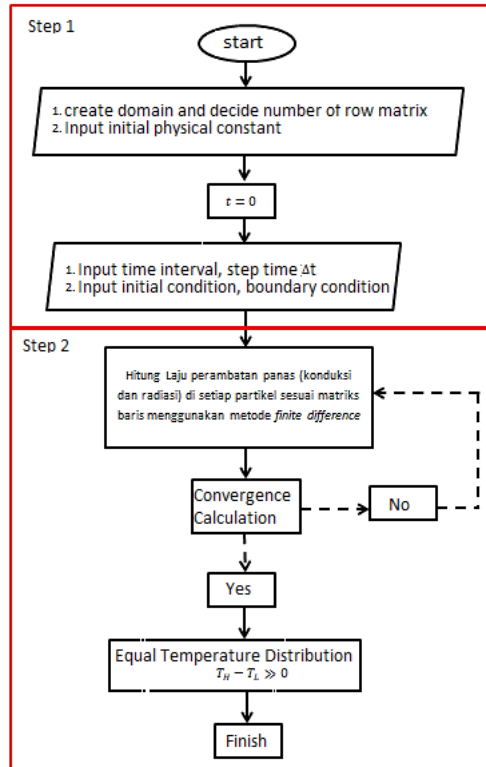


Figure 3. Flow chart of calculation 1-Dimension heat propagation rate

As shown in Figure 3, the numerical scheme is divided into two stages, namely: (1) creating a computational domain, and (2) calculating the heat distribution. The following are parameter values for the characteristics of some of the materials used:

Table 1. The constant composition of materials layer (Tarney 1989)

Material	$\kappa$ (W·m <sup>-1</sup> ·K <sup>-1</sup> )	$c_p$ (J·kg <sup>-1</sup> ·K <sup>-1</sup> )	$\epsilon$
Stainless steel skillet [4]	14,9	477	0,17
Sand [4]	0,27	1515	0,9
Peanuts	0,168 [5]	2230 [5]	0,8

### 3.3 Computational Results

The initial step divided layers of roasted composition based on conduction and radiation events that occur in each layer. The conduction layer uses equation (3), while the radiation layer uses equation (4). In equations (3) and (4), each variable sought is the amount of heat energy generated from the conduction and radiation process. However, in the equation, the highest temperature and lowest temperature variables are known. In this study, what is sought is the temperature distribution in each layer of the roasted composition so that the variable sought from the equation becomes the lowest temperature. The description of the lowest temperature variable sought at each layer is clarified in Figure 2.

The next step was to determine the reduction of the intensity parameters in each layer of the roasted composition. The intensity parameter in equation (7) is useful for knowing how much heat energy is emitted per unit area and time on each surface of the layer. The intensity

value is given the same at each layer. The limitation to note is that the difference between the highest temperature and the lowest temperature must be greater than zero, which is formulated by:  $T_H - T_L \gg 0$ . The intensity parameter value used is  $I = 1 \text{ J}\cdot\text{s}\cdot\text{m}^{-2}$ ,  $\sigma = 5,670367 \times 10^{-8} \text{ W}\cdot\text{m}^{-2}\cdot\text{K}^{-4}$ ,  $\Delta t = 10^{-5} \text{ s}$ ,  $L = 1 \text{ mm} = 10^{-3} \text{ m}$ ,  $V = 220 \text{ V}$ ,  $R = 10 \Omega$ ,  $T_{env} = 300 \text{ K}$ .

The completion of the temperature distribution at each layer of the roasting process used the row matrix method. Each layer was given a number notation like Figure 4. This method was done by making a supposition whether each layer occurs in the process of conduction or radiation. Each number notation was entered in the formula in Figure 4 to find out the temperature in each layer.



Figure 4. Number notation for row matrix at each layer of the roast process composition (-1: heater; 0: air in the vacuum chamber; 1: sand; 2: food/roast object).

Using empirical data in Table 1, the initial temperature in each roasting layer was 300 K. During an interval of 3.62 seconds, a temperature rose not far from 300 K for layers far from the heat source. The temperature was almost evenly distributed due to conduction and radiation. The highest temperature of 409.4 K at 3.62 seconds was found in the pan layer directly facing the heat source (Figure 5).

		Materials (-1: heater .0: vakum, 1: sand 2: food)															
		-1	1	0	1	1	0	1	2	2	2	1	0	1	1	1	0
Time	0.00	300.00	300.00	300.00	300.00	300.00	300.00	300.00	300.00	300.00	300.00	300.00	300.00	300.00	300.00	300.00	300.00
	0.16	304.84	304.83	300.05	300.05	300.05	300.04	300.04	300.04	300.03	300.02	300.02	300.04	300.04	300.04	300.04	300.04
	0.45	313.60	313.60	300.13	300.13	300.13	300.12	300.12	300.12	300.11	300.10	300.10	300.12	300.12	300.12	300.12	300.12
	0.62	318.74	318.73	300.19	300.19	300.19	300.17	300.17	300.16	300.15	300.15	300.15	300.17	300.17	300.17	300.17	300.17
	0.92	327.80	327.80	300.30	300.30	300.30	300.25	300.25	300.24	300.24	300.23	300.23	300.25	300.25	300.25	300.25	300.25
	1.02	330.82	330.82	300.34	300.34	300.34	300.28	300.28	300.27	300.26	300.26	300.26	300.28	300.28	300.28	300.28	300.28
	1.29	339	339	300.5	300.5	300.5	300.4	300.4	300.4	300.3	300.3	300.3	300.4	300.4	300.4	300.4	300.4
	1.43	343.2	343.2	300.5	300.5	300.5	300.4	300.4	300.4	300.4	300.4	300.4	300.4	300.4	300.4	300.4	300.4
	1.70	351.4	351.4	300.7	300.7	300.7	300.5	300.5	300.5	300.5	300.4	300.4	300.5	300.5	300.5	300.5	300.5
	1.95	358.9	358.9	300.8	300.8	300.8	300.5	300.5	300.5	300.5	300.5	300.5	300.5	300.5	300.5	300.5	300.5
	2.20	366.5	366.5	300.9	300.9	300.9	300.6	300.6	300.6	300.6	300.6	300.6	300.6	300.6	300.6	300.6	300.6
	2.67	380.7	380.7	301.2	301.2	301.2	300.7	300.7	300.7	300.7	300.7	300.7	300.7	300.7	300.7	300.7	300.7
	2.67	380.7	380.7	301.2	301.2	301.2	300.7	300.7	300.7	300.7	300.7	300.7	300.7	300.7	300.7	300.7	300.7
	3.14	394.9	394.9	301.6	301.6	301.6	300.9	300.9	300.9	300.9	300.8	300.8	300.9	300.9	300.9	300.9	300.9
3.26	398.5	398.5	301.7	301.7	301.7	300.9	300.9	300.9	300.9	300.9	300.9	300.9	300.9	300.9	300.9	300.9	
3.42	403.4	403.4	301.8	301.8	301.8	300.9	300.9	300.9	300.9	300.9	300.9	300.9	300.9	300.9	300.9	300.9	
3.55	407.3	407.3	302	302	302	301	301	301	301	301	301	301	301	301	301	301	
3.62	409.4	409.4	302	302	302	301	301	301	301	301	301	301	301	301	301	301	

Figure 5. Temperature distribution in each layer composition of the roasting process

#### 4. Conclusions

From the simulation of 1-Dimension heat propagation process, one-dimensional modeling of the heat propagation of granular material in the roasting process can be formulated and the heat propagation modeled in this study was unidirectional. From the conclusions of the results of these studies, some things recommended for this study or further research are the heat intensity parameter factors still need to be verified through experiments, so the results will be free of numerical parameters, and the development of research with 2-Dimensional heat propagation modeling will be more interesting because it will add to its complexity.

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